# Mathematics

# LEBANESE UNIVERSITY

## Faculty of Medical Sciences

Date : October 13, 2009

Time : 30 minutes

## Entrance Exam to 2nd Year (2009-2010)

## Subject : Mathematics

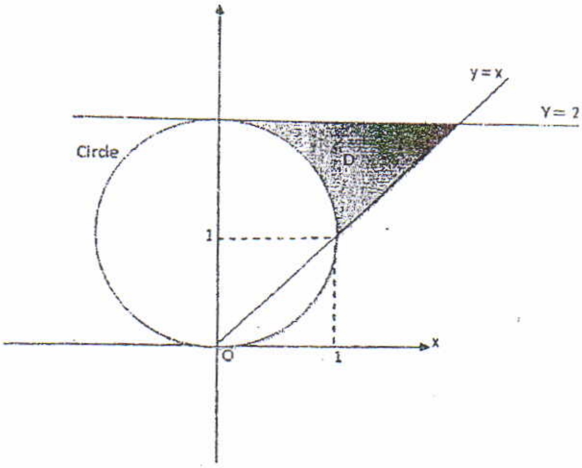
1- (7 points)

Let $f(x) = \sqrt[3]{x^3 + x^2 + 1}\ e^{1/x}$.

Using finite expansions, find the equation of the oblique asymptote (d) to the representative curve (C) of $f$ at $+\infty$ and precise their relative position.

2- (8 points)

Consider $I = \iint_D f(x,y)dxdy$.



a. Using Cartesian coordinates, write the double integral $I$ in two different forms.
b. Using polar coordinates calculate the area of the domain D.

3- (5 points)

Calculate $\int \frac{t^2}{2t^2+1} dt$. Then deduce $\int \frac{\tan^2(x)}{1+\sin^2 x} dx$.

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LEBANESE UNIVERSITY
Faculty of Medical Sciences

Date: 8/10/2008
**Duration:** 1 hour

## Entrance exam for the 2nd year
## Academic year 2008-2009
## Subject: Mathematics
## 1st part: STATISTICS

I- A statistical study about the relationship between the two variables X and Y has given the following results:

- The regression line of Y on X: $Y = 0.4X + 0.2$
- The regression line of X on Y: $X = a'Y + 0.4$
- The variance of X is equal to 4, the variance of Y is equal to 1.

Determine a' and $\overline{X}$. (5 pts)

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II- In a population :

- The probability that a smoker has the disease M is equal to 0.8
- The probability that a non somker has the disease M is equal to 0.3
- The probability for an individual to have the disease M and to be a smoker is equal to 0.2.

Compute the probability that an individual of this population has the disease M. (5 pts)

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III- In a city A, the probability that an individual has flu is equal to 0.8. In a city B, the probability that an individual has flu is equal to 0.9.
From A, a random sample of 100 persons is selected, and also from B, a random sample of 100 persons is selected.
What is the probability to find in the set formed by the two samples a number of persons with flu included between 160 and 180? (6 pts)

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IV- In a population of men, the weight of a man (in kg) is a random variable with mean m and standard deviation of 5.
At what risk, can m be estimated by a confidence interval of width 2.24 kg based on a sample of size n=100 men? (4 pts)

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## $2^{nd}$ Part: Analysis

**Exercise 1: (7 points)**

Consider the function $f(x)=\dfrac{\ln(1+x+x^2)+e^{ax^2}}{1+x^2}$, where $a$ is a real parameter.

1. Give the finite expansion of $f$ to order 3 at $x=0$.
2. Let g be the function whose finite expansion to order 3 at $x=0$ is given by:

$$g(x)=1+x+\left(a-\frac{1}{2}\right)x^2-\frac{5}{3}x^3+x^3\varepsilon(x).$$

   a. Find the equation of the tangent line (T) to the curve (C) of $g$ at $x=0$.
   b. Discuss according to the values of the parameter $a$ the relative position of (T) and (C).

**Exercise 2: (6 points)**

Given the following double integral $I=\int_0^1\int_{\sqrt{y}}^1 e^{\frac{y}{x}}dxdy$.

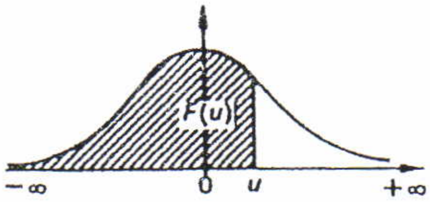
1. Sketch the domain of integration $D$.
2. Calculate $I$.

**Exercise 3: (7 points)**

Calculate the following integral: $I=\int e^x \ln\left(\dfrac{e^x}{1+e^x}\right)dx$.

# Standard Normal Distribution

## $F(u) = P[Z \leq u]\ (u \geq 0)$



| u | 0,00 | 0,01 | 0,02 | 0,03 | 0,04 | 0,05 | 0,06 | 0,07 | 0,08 | 0,09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0,0 | 0,500 0 | 0,504 0 | 0,508 0 | 0,512 0 | 0,516 0 | 0,519 9 | 0,523 9 | 0,527 9 | 0,531 9 | 0,535 9 |
| 0,1 | 0,539 8 | 0,543 8 | 0,547 8 | 0,551 7 | 0,555 7 | 0,559 6 | 0,563 6 | 0,567 5 | 0,571 4 | 0,575 3 |
| 0,2 | 0,579 3 | 0,583 2 | 0,587 1 | 0,591 0 | 0,594 8 | 0,598 7 | 0,602 6 | 0,606 4 | 0,610 3 | 0,614 1 |
| 0,3 | 0,617 9 | 0,621 7 | 0,625 5 | 0,629 3 | 0,633 1 | 0,636 8 | 0,640 6 | 0,644 3 | 0,648 0 | 0,651 7 |
| 0,4 | 0,655 4 | 0,659 1 | 0,662 8 | 0,666 4 | 0,670 0 | 0,673 6 | 0,677 2 | 0,680 8 | 0,684 4 | 0,687 9 |
| 0,5 | 0,691 5 | 0,695 0 | 0,698 5 | 0,701 9 | 0,705 4 | 0,708 8 | 0,712 3 | 0,715 7 | 0,719 0 | 0,722 4 |
| 0.6 | 0,725 7 | 0,729 0 | 0,732 4 | 0,735 7 | 0,738 9 | 0,742 2 | 0,745 4 | 0,748 6 | 0,751 7 | 0,754 9 |
| 0,7 | 0,758 0 | 0,761 1 | 0,764 2 | 0,767 3 | 0,770 4 | 0,773 4 | 0,776 4 | 0,779 4 | 0,782 3 | 0,785 2 |
| 0,8 | 0,788 1 | 0,791 0 | 0,793 9 | 0,796 7 | 0,799 5 | 0,802 3 | 0,805 1 | 0,807 8 | 0,810 6 | 0,813 3 |
| 0,9 | 0,815 9 | 0,818 6 | 0,821 2 | 0,823 8 | 0,826 4 | 0,828 9 | 0,831 5 | 0,834 0 | 0,836 5 | 0,838 9 |
| 1,0 | 0,841 3 | 0,843 8 | 0,846 1 | 0,848 5 | 0,850 8 | 0,853 1 | 0,855 4 | 0,857 7 | 0,859 9 | 0,862 1 |
| 1,1 | 0,864 3 | 0,866 5 | 0,868 6 | 0,870 8 | 0,872 9 | 0,874 9 | 0,877 0 | 0,879 0 | 0,881 0 | 0,883 0 |
| 1,2 | 0,884 9 | 0,886 9 | 0,888 8 | 0,890 7 | 0,892 5 | 0,894 4 | 0,896 2 | 0,898 0 | 0,899 7 | 0,901 5 |
| 1,3 | 0,903 2 | 0,904 9 | 0,906 6 | 0,908 2 | 0,909 9 | 0,911 5 | 0,913 1 | 0,914 7 | 0,916 2 | 0,917 7 |
| 1,4 | 0,919 2 | 0,920 7 | 0,922 2 | 0,923 6 | 0,925 1 | 0,926 5 | 0,927 9 | 0,929 2 | 0,930 6 | 0,931 9 |
| 1,5 | 0,933 2 | 0,934 5 | 0,935 7 | 0,937 0 | 0,938 2 | 0,939 4 | 0,940 6 | 0,941 8 | 0,942 9 | 0,944 1 |
| 1,6 | 0,945 2 | 0,946 3 | 0,947 4 | 0,948 4 | 0,949 5 | 0,950 5 | 0,951 5 | 0,952 5 | 0,953 5 | 0,954 5 |
| 1,7 | 0,955 4 | 0,956 4 | 0,957 3 | 0,958 2 | 0,959 1 | 0,959 9 | 0,960 8 | 0,961 6 | 0,962 5 | 0,963 3 |
| 1,8 | 0,964 1 | 0,964 9 | 0,965 6 | 0,966 4 | 0,967 1 | 0,967 8 | 0,968 6 | 0,969 3 | 0,969 9 | 0,970 6 |
| 1,9 | 0,971 3 | 0,971 9 | 0,972 6 | 0,973 2 | 0,973 8 | 0,974 4 | 0,975 0 | 0,975 6 | 0,976 1 | 0,976 7 |
| 2,0 | 0,977 2 | 0,977 9 | 0,978 3 | 0,978 8 | 0,979 3 | 0,979 8 | 0,980 3 | 0,980 8 | 0,981 2 | 0,981 7 |
| 2,1 | 0,982 1 | 0,982 6 | 0,983 0 | 0,983 4 | 0,983 8 | 0,984 2 | 0,984 6 | 0,985 0 | 0,985 4 | 0,985 7 |
| 2,2 | 0,986 1 | 0,986 4 | 0,986 8 | 0,987 1 | 0,987 5 | 0,987 8 | 0,988 1 | 0,988 4 | 0,988 7 | 0,989 0 |
| 2,3 | 0,989 3 | 0,989 6 | 0,989 8 | 0,990 1 | 0,990 4 | 0,990 6 | 0,990 9 | 0,991 1 | 0,991 3 | 0,991 6 |
| 2,4 | 0,991 8 | 0,992 0 | 0,992 2 | 0,992 5 | 0,992 7 | 0,992 9 | 0,993 1 | 0,993 2 | 0,993 4 | 0,993 6 |
| 2,5 | 0,993 8 | 0,994 0 | 0,994 1 | 0,994 3 | 0,994 5 | 0,994 6 | 0,994 8 | 0,994 9 | 0,995 1 | 0,995 2 |
| 2,6 | 0,995 3 | 0,995 5 | 0,995 6 | 0,995 7 | 0,995 9 | 0,996 0 | 0,996 1 | 0,996 2 | 0,996 3 | 0,996 4 |
| 2,7 | 0,996 5 | 0,996 6 | 0,996 7 | 0,996 8 | 0,996 9 | 0,997 0 | 0,997 1 | 0,997 2 | 0,997 3 | 0,997 4 |
| 2,8 | 0,997 4 | 0,997 5 | 0,997 6 | 0,997 7 | 0,997 7 | 0,997 8 | 0,997 9 | 0,997 9 | 0,998 0 | 0,998 1 |
| 2,9 | 0,998 1 | 0,998 2 | 0,998 2 | 0,998 3 | 0,998 4 | 0,998 4 | 0,998 5 | 0,998 5 | 0,998 6 | 0,998 6 |

| u | 3,0 | 3,1 | 3,2 | 3,3 | 3,4 | 3,5 | 3,6 | 3,8 | 4,0 | 4,5 |
|---|---|---|---|---|---|---|---|---|---|---|
| F(u) | 0,998 65 | 0,999 04 | 0,999 31 | 0,999 52 | 0,999 66 | 0,999 76 | 0,999 841 | 0,999 928 | 0,999 968 | 0,999 997 |

LEBANESE UNIVERSITY
**Faculty of Medicine,**
**Faculty of Pharmacy**
**Faculty of Dentistry**

**Date: 21/9/2010**
**Time: 30 minutes**

## Entrance exam to 2nd year (2010-2011)
## Subject: Mathematics

I. (6 pts) Let $a$ be a given real number.

1- a- Give the finite expansions to order 3 in the neighborhood of $x = 0$, of

$$\cos ax, \quad e^{x-1+\cos ax}, \quad \sin x \quad \text{and} \quad \ln(1+\sin x).$$

b. Deduce the finite expansion to order 3 in the neighborhood of $x = 0$ of the function

$$f(x) = e^{x-1+\cos ax} + \ln(1+\sin x).$$

2- Let g be the function whose finite expansion to order 3 near $x = 0$ is given by:

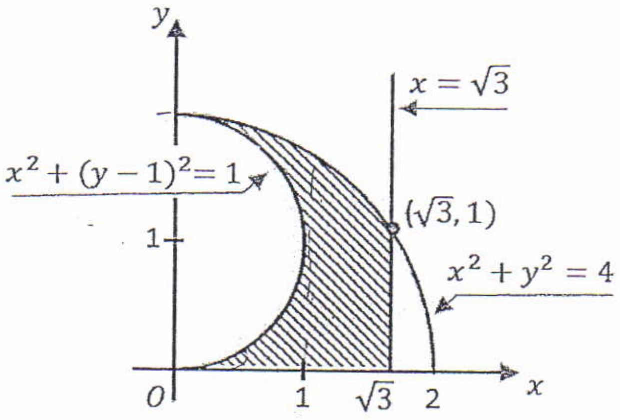
$$g(x) = 1 + 2x - \frac{a^2}{2}x^2 + \left(\frac{1}{3} - \frac{a^2}{2}\right)x^3 + x^3\varepsilon(x)$$

a- Write the equation of the tangent to the representative curve (C) of the function $g$ at $x = 0$.

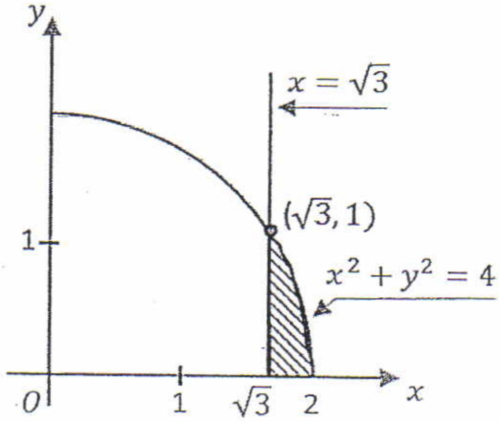
b- Discuss according to the values of the parameter $a$ the relative position of (C) with respect to this tangent line near $x = 0$.

II. (8 pts) 1- Let D be the shaded domain shown in the figure.

Using Cartesian coordiantes write down $\iint_D f(x,y)dxdy$ in two different ways.



2- Using polar coordiantes calculate the area of the shaded domain Δ, shown in the second figure.



III. (6 pts) 1-Calculate the integral :

$$\int \frac{dx}{3 + \cos x}$$

2- Solve the differential equation $xy' - y = x\left(3 + \cos\frac{y}{x}\right)$.

**Ex 1** 1a) Au voisinage de 0 à l'ordre 3:

$\frac{1}{2}+\frac{1}{2}$ $\boxed{\cos(ax) = 1 - \frac{a^2}{2}x^2 + x^3\varepsilon(x)}$

$$e^{x-1+\cos ax} = e^{x - a^2/2\, x^2 + x^3\varepsilon(x)}$$

$\frac{1}{2}$ $= 1 + x - a^2/2\, x^2 + \frac{1}{2}\left(x - \frac{a^2}{2}x^2\right)^2 + \frac{1}{6}\left(x - \frac{a^2}{2}x^2\right)^3 + x^3\varepsilon($

$\frac{1}{2}$ $= 1 + x - \frac{a^2}{2}x^2 + \frac{1}{2}x^2 - \frac{a^2}{2}x^3 + \frac{1}{6}x^3 + x^3\varepsilon(x)$

$$\boxed{e^{x-1+\cos ax} = 1 + x + \frac{1}{2}(1-a^2)x^2 + \left(\frac{1}{6} - a^2/2\right)x^3 + x^3\varepsilon(x)}$$

$\frac{1}{2}$ $\boxed{\sin x = x - \frac{x^3}{6} + x^3\varepsilon(x)}$

$$\ln(1+\sin x) = \ln\left(1 + x - \frac{x^3}{6}\right) + x^3\varepsilon(x)$$

$\frac{1}{2}$ $= x - \frac{x^3}{6} - \frac{1}{2}\left(x - \frac{x^3}{6}\right)^2 + \frac{1}{3}\left(x - \frac{x^3}{6}\right)^3 + x^3\varepsilon(x$

$\frac{1}{2}$ $= x - \frac{x^3}{6} - \frac{x^2}{2} + \frac{1}{3}x^3 + x^3\varepsilon(x)$

$$\boxed{\ln(1+\sin x) = x - \frac{x^2}{2} + \frac{1}{6}x^3 + x^3\varepsilon(x)}$$

$\frac{1}{2}$

b) $f(x) = 1 + x + \frac{1}{2}(1-a^2)x^2 + \left(\frac{1}{6} - a^2/2\right)x^3 + x - \frac{x^2}{2} + \frac{1}{6}x^3 + x^3\varepsilon(x)$

$$\boxed{f(x) = 1 + 2x - \frac{a^2}{2}x^2 + \left(\frac{1}{3} - \frac{a^2}{2}\right)x^3 + x^3\varepsilon(x)}$$

tgt $\frac{1}{2}$

2) a) $y = 1 + 2x$

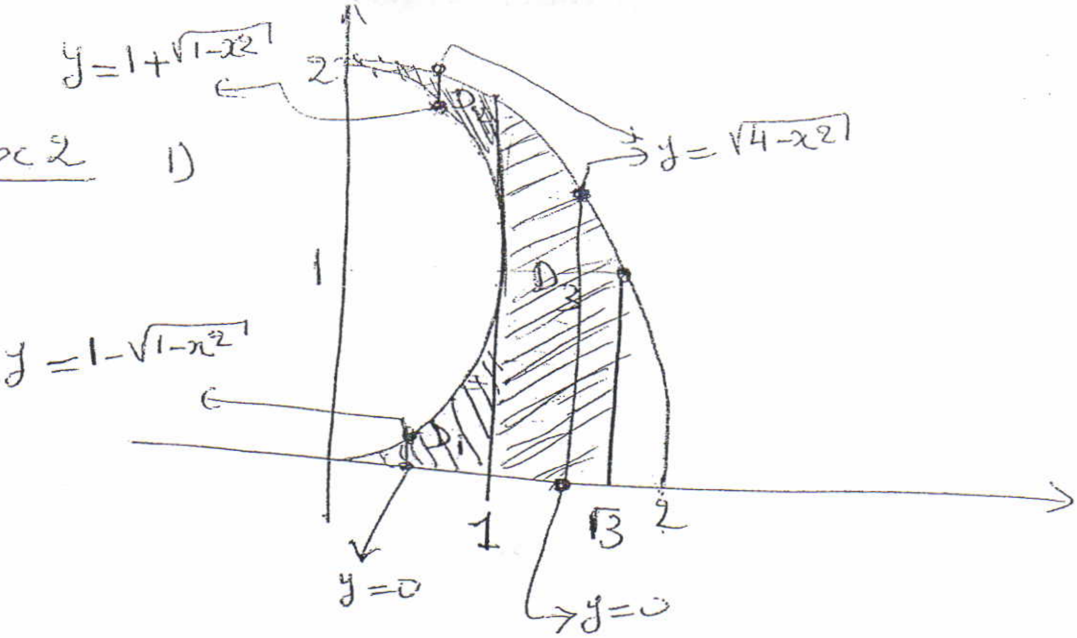
$\frac{1}{2}$ b) *) si $a \neq 0$ ; $f(x) - y \underset{0}{\simeq} -\frac{a^2}{2} < 0 \Rightarrow$ $C_f$ est en dessous de la tgte

*) si $a = 0$ ; $f(x) - y \underset{0}{\simeq} \frac{1}{3}x^3$

$\frac{1}{2}$ si $x > 0$ ; $C_f$ est au-dessus de la tgte

$\frac{1}{2}$ si $x < 0$ ; $C_f$ est au-dessous de la tgte.

**Ex 2** 1)



**En fixant x :** $D = D_1 \cup D_2 \cup D_3$

$$D_1 : \begin{cases} 0 \leq x \leq 1 \\ 0 \leq y \leq 1-\sqrt{1-x^2} \end{cases}$$

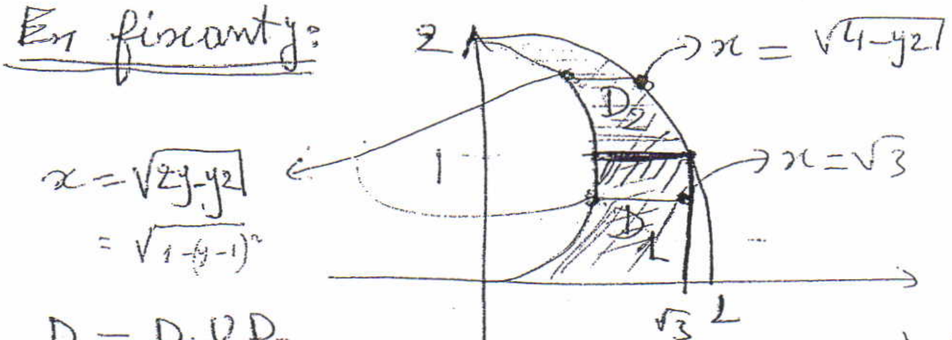
$$D_2 : \begin{cases} 0 \leq x \leq 1 \\ 1+\sqrt{1-x^2} \leq y \leq \sqrt{4-x^2} \end{cases}$$

$\frac{1}{2} \times 10$

$$D_3 : \begin{cases} 1 \leq x \leq \sqrt{3} \\ 0 \leq y \leq \sqrt{4-x^2} \end{cases}$$

$$I = \int_0^1 dx \int_0^{1-\sqrt{1-x^2}} f\,dy + \int_0^1 dx \int_{1+\sqrt{1-x^2}}^{\sqrt{4-x^2}} f\,dy + \int_1^{\sqrt{3}} dx \int_0^{\sqrt{4-x^2}} f\,dy$$

**En fixant y :**



$D = D_1 \cup D_2$

$$D_1 : \begin{cases} 0 \leq y \leq 1 \\ \sqrt{2y-y2} \leq x \leq \sqrt{3} \end{cases} \qquad \sqrt{1-(y-1)^2} \leftarrow$$

$$D_2 : \begin{cases} 1 \leq y \leq 2 \\ \sqrt{2y-y2} \leq x \leq \sqrt{4-y2} \end{cases}$$

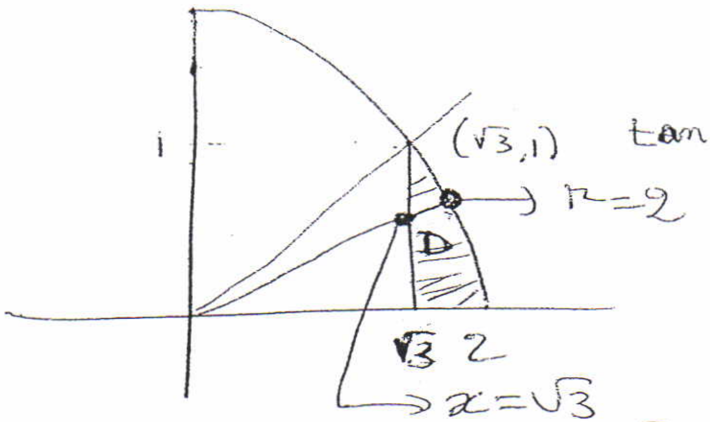
$$I = \int_0^1 dy \int_{\sqrt{2y-y2}}^{\sqrt{3}} f\,dx + \int_1^2 dy \int_{\sqrt{2y-y2}}^{\sqrt{4-y2}} f\,dx$$

$\sqrt{2y-y2} = \sqrt{1-(y-1)^2}$

$\sqrt{2y-y2} = \sqrt{1-(y-1)}$

Ex2 2)

$$D : \begin{cases} 0 \leq \theta \leq \pi/6 \\ \frac{\sqrt{3}}{\cos\theta} \leq r \leq 2 \end{cases}$$



$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \pi/6$

$r\cos\theta = \sqrt{3} \Rightarrow r = \frac{\sqrt{3}}{\cos\theta}$

fle $\frac{1}{2}$

$$\text{aire }(D) = \iint_D dx\,dy = \iint_\Delta r\,dr\,d\theta$$

$$= \int_0^{\pi/6} d\theta \int_{\frac{\sqrt{3}}{\cos\theta}}^{2} r\,dr$$

$$= \frac{1}{2}\int_0^{\pi/6} \left[r^2\right]_{\frac{\sqrt{3}}{\cos\theta}}^{2} d\theta$$

Calcul $\frac{1}{2}$

$$= \frac{1}{2}\int_0^{\pi/6} \left(4 - \frac{3}{\cos^2\theta}\right) d\theta$$

$$= \frac{1}{2}\left[4\theta - 3\tan\theta\right]_0^{\pi/6}$$

$$= \frac{1}{2}\left[4\pi/6 - 3\cdot\frac{1}{\sqrt{3}}\right]$$

$$= \pi/3 - \frac{\sqrt{3}}{2}$$

EX III

1 - $\int \frac{dx}{3+\cos x}$

$\frac{1}{2}$ $\tan\frac{x}{2} = t$, $dx = \frac{1}{2}\ \frac{2\,dt}{1+t^2}$, $\frac{1}{2}\cos x = \frac{1-t^2}{1+t^2}$

$$\rightarrow \int \frac{\frac{2\,dt}{1+t^2}}{3+\frac{1-t^2}{1+t^2}} = \int \frac{2\,dt}{3+3t^2+1-t^2} = \int \frac{2\,dt}{4+2t^2} \left(\frac{1}{2}\right)$$

$$① = \int \frac{dt}{2+t^2} = \frac{1}{2}\int \frac{dt}{1+\left(\frac{t}{\sqrt{2}}\right)^2} = \frac{1\cdot\sqrt{2}}{2}\int \frac{d\left(t/\sqrt{2}\right)}{1+\left(\frac{t}{\sqrt{2}}\right)^2}$$

$$= \frac{\sqrt{2}}{2}\arctan\left(\frac{t}{\sqrt{2}}\right) + C = \frac{\sqrt{2}}{2}\arctan\left(\frac{\tan x/2}{\sqrt{2}}\right) + C$$

$t \rightarrow x$ $\left(\frac{1}{2}\right)$

2 - $y' - \frac{y}{x} = 3 + \cos\frac{y}{x}$

$\left(\frac{1}{2}\right)$ $\frac{y}{x} = z$, $y = xz$ on a $y' = z + xz'$ $\left(\frac{1}{2}\right)$

$z + xz' - z = 3 + \cos z$,

$\left(\frac{1}{2}\right) \rightarrow x\frac{dz}{dx} = 3 + \cos z$,

$\left(\frac{1}{2}\right)$ $\int \frac{dx}{x} = \int \frac{dz}{3+\cos z}$

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$\left(\frac{1}{2}\right)$ $\ln\frac{x}{C} = \frac{\sqrt{2}}{2}\arctan\left(\frac{\tan\frac{z}{2}}{\sqrt{2}}\right)$ $\left(\frac{1}{2}\right)$,

$$\boxed{\ln\frac{x}{C} = \frac{\sqrt{2}}{2}\arctan\left(\frac{\tan\frac{y}{2x}}{\sqrt{2}}\right)}$$

$\left(-\frac{1}{2}\right)$

Vrai

$$\int \frac{dt}{a^2+a t^2} = \frac{1\,\cancel{a}}{\sqrt{a}}\arctan at$$

$$\int \frac{dt}{1+a^2t^2} = \frac{a}{a}\int \frac{dt}{1+(at)^2}$$

$$= \frac{1}{a}\int \frac{dv}{1+v^2} = \frac{1}{a}\arctan$$

LEBANESE UNIVERSITY
Faculty of Medical sciences
Faculty of Pharmacy
Faculty of Dentistry

13 September 2011
Duration : 30 minutes

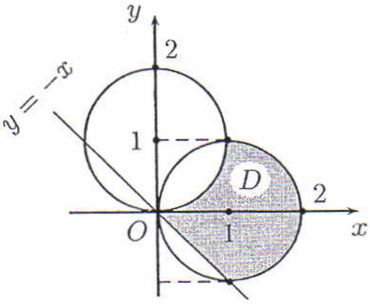
Entrance Exam To The II Year
Academic year 2011-2012
Mathematics

## Exercise 1 [7 pts]

1° Write in polar coordinates the following double integral

$$I = \iint_D f(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

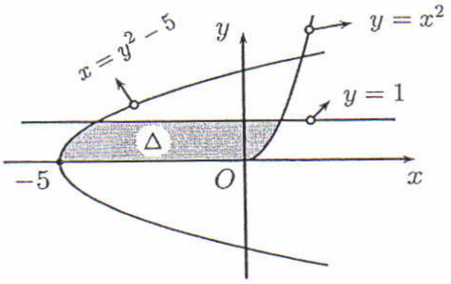
where $D$ is the shaded domain shown in the adjacent figure.



2° Using cartesian coordinates, write down the following double integral, in two different ways

$$I = \iint_\Delta f(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

where $\Delta$ is the shaded domain, shown in the second figure.



## Exercise 2 [7 pts]

Let $a$ be a real parameter.

1° Give the finite expansion of order 3 in neighborhood of $t=0$ of

$$g(t) = e^t + a\ln(1+t) - \sin(at).$$

2° Deduce the equation of the oblique asymptote at $+\infty$ to the representative curve $(C)$ of

$$f(x) = x\left[e^{\frac{1}{x}} + a\ln\left(\frac{1+x}{x}\right) - \sin\left(\frac{a}{x}\right)\right].$$

Discuss, according to the values of $a$, the relative position of $(C)$ and its asymptote near $+\infty$.

**Exercise 3** [6 pts]

1° Calculate the following integral: $\int \cos x \ln(1-\cos x)\,\mathrm{d}x$.

2° (a) Calculate $I(t) = \int \frac{t^2-1}{t^2+1}\,\mathrm{d}t$.

(b) Deduce $J(x) = \int \frac{\cos^3 x}{1+\sin^2 x}\,\mathrm{d}x$.

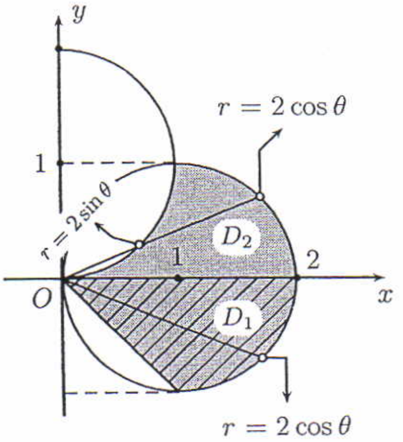
## Exercise 1

1° In polar coordinates, the domain $D$ is defined by

$$D = D_1 \cup D_2,$$

where $D_1 : \begin{cases} -\frac{\pi}{4} \le \theta \le 0 \\ 0 \le r \le 2\cos\theta \end{cases}$

and, $D_2 : \begin{cases} 0 \le \theta \le \frac{\pi}{4} \\ 2\sin\theta \le r \le 2\cos\theta \end{cases}$

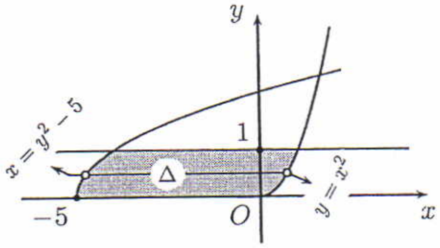


Thus

$$I = \int_{-\frac{\pi}{4}}^{0} \mathrm{d}\theta \int_{0}^{2\cos\theta} f(r\cos\theta, r\sin\theta)\, r\, \mathrm{d}r + \int_{0}^{\frac{\pi}{4}} \mathrm{d}\theta \int_{2\sin\theta}^{2\cos\theta} f(r\cos\theta, r\sin\theta)\, r\, \mathrm{d}r.$$

2° **First way**: If $y$ is held fixed (horizontally), then $\Delta$ is defined by

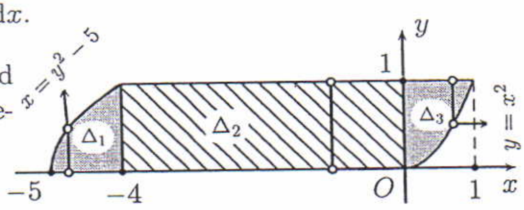
$$\Delta : \begin{cases} 0 \le y \le 1 \\ y^2 - 5 \le x \le \sqrt{y} \end{cases}$$



Hence $I = \int_0^1 \mathrm{d}y \int_{y^2-5}^{\sqrt{y}} f(x, y)\, \mathrm{d}x.$

**Second way**: Now if $x$ is held fixed (vertically), then $\Delta$ is defined by:

$$\Delta = \Delta_1 \cup \Delta_2 \cup \Delta_3,$$



where

$\Delta_1 : \begin{cases} -5 \le x \le -4 \\ 0 \le y \le \sqrt{x+5} \end{cases}$ $\Delta_2 : \begin{cases} -4 \le x \le 0 \\ 0 \le y \le 1 \end{cases}$ $\Delta_3 : \begin{cases} 0 \le x \le 1 \\ x^2 \le y \le 1 \end{cases}$

Hence,

$$I = \int_{-5}^{-4} \mathrm{d}x \int_{0}^{\sqrt{x+5}} f(x, y)\, \mathrm{d}y + \int_{-4}^{0} \mathrm{d}x \int_{0}^{1} f(x, y)\, \mathrm{d}y + \int_{0}^{1} \mathrm{d}x \int_{x^2}^{1} f(x, y)\, \mathrm{d}y.$$

## Exercise 2

1° In neighborhood of $t = 0$ and to order 3, we have

$$g(t) = 1 + t + \frac{t^2}{2} + \frac{t^3}{6} + a\left(t - \frac{t^2}{2} + \frac{t^3}{3}\right) - \left(at - \frac{a^3t^3}{6}\right) + t^3\varepsilon(t).$$

And consequently,

$$g(t) = 1 + t + \frac{1-a}{2}t^2 + \frac{1+2a+a^3}{6}t^3 + t^3\varepsilon(t).$$

2° Let $x = \frac{1}{t} \Rightarrow$ if $t \to 0_+$, $x \to +\infty$. Then

$$tf\left(\frac{1}{t}\right) = g(t) = 1 + t + \frac{1-a}{2}t^2 + \frac{1+2a+a^3}{6}t^3 + t^3\varepsilon(t),$$

and thus

$$f(x) = x + 1 + \left(\frac{1-a}{2}\right)\frac{1}{x} + \left(\frac{1+2a+a^3}{6}\right)\frac{1}{x^2} + \frac{1}{x^2}\varepsilon(x).$$

$y = x + 1$ is the equation of the oblique asymptote to $(C)$ at $+\infty$.

- For $a \neq 1$; $f(x) - y \underset{+\infty}{\simeq} \frac{1-a}{2}\frac{1}{x}$, thus
  - if $a < 1$, then the curve $(C)$ is above the asymptote;
  - if $a > 1$, then the curve $(C)$ is below the asymptote.
- For $a = 1$; $f(x) - y \underset{+\infty}{\simeq} \frac{1+2a+a^3}{6}\frac{1}{x^2} = \frac{2}{3x^2} > 0$, then the curve $(C)$ is above the asymptote.

## Exercise 3

1° Integrating by parts, we set

$$u = \ln(1 - \cos x) \quad \text{and} \quad dv = \cos x \, dx$$

thus $du = \frac{\sin x}{1 - \cos x} dx$ and $v = \sin x$.

Therefore,

$$\begin{aligned}\int \cos x \ln(1 - \cos x) &= \sin x \ln(1 - \cos x) - \int \frac{\sin^2 x}{1 - \cos x} dx \\ &= \sin x \ln(1 - \cos x) - \int \frac{1 - \cos^2 x}{1 - \cos x} dx \\ &= \sin x \ln(1 - \cos x) - \int (1 + \cos x) dx \\ &= \sin x \ln(1 - \cos x) - x - \sin x + \text{cnst}.\end{aligned}$$

2° (a) $$\int \frac{t^2-1}{t^2+1}\,\mathrm{d}t = \int \frac{(t^2+1)-2}{t^2+1}\,\mathrm{d}t = \int \mathrm{d}t - 2\int \frac{\mathrm{d}t}{1+t^2}$$
$$= t - 2\arctan t + \text{cnst}.$$

(b) Let $t = \sin x$,

$$\int \frac{\cos^3 x}{1+\sin^2 x}\,\mathrm{d}x = \int \frac{1-t^2}{1+t^2}\,\mathrm{d}t = 2\arctan t - t + \text{cnst}.$$

Therefore, $\displaystyle\int \frac{\cos^3 x}{1+\sin^2 x}\,\mathrm{d}x = 2\arctan(\sin x) - \sin x + \text{cnst}.$