1-(2 points)

In the table below, only one of the proposed answers to each question is correct.
Write down the number of each question and give, **with justification,** the answer corresponding to it.

N°	Questions	Answers a	Answers b	Answers c	Answers d
1	If $S_n = 1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2n}$ then $\lim\limits_{n \to +\infty} S_n =$	4	2	$+\infty$	8
2	The solution of the equation $\ln(x-1) + \ln(x+2) = 2\ln 2$ is:	{2;-3}	{2;3}	{2}	{1;2}
3	If $C_n^7 = C_n^5$ then n=	35	12	14	10
4	Given two events A and B. If P(B) = 0.2 and P(A/B) = 0.6 then P(A∩B)=	0.12	0.4	0.8	0.2

II- (5 points)

Given two urns U and V.
U contains **four** white balls numbered from 1 to 4 and **three** red balls numbered from 5 to 7.
V contains **three** white balls numbered from 1 to 3 and two red balls numbered from 4 to 5.

1) We draw randomly one ball from each urn.
Calculate the probability of each of the following events: R : « the two drawn balls are red »
C : « the two drawn balls have the same colour »
N : « the two drawn balls carry the same number ».

2) The **twelve** balls in urns U and V are emptied into an urn W.
We draw simultaneously and randomly **three** balls from the urn W.

a- What is the probability of obtaining the ball carrying the number 7 among the three drawn balls?
b- What is the probability of obtaining the two balls carrying the number 4 among the three drawn balls?
c- What is the probability of drawing three balls carrying numbers that can form the number 226?
d- Knowing that the three drawn balls are red, what is the probability that two of these drawn balls carry the number 5?

III- (5 points)

A club had 1000 members in January 2004.
It is known that during the month of January of every year, 25% of the members leave the club while 100 new members join those who remain in the club.
Designate by U_n the number of members in this club in January of the year $(2004 + n)$.

1) Justify that $U_{n+1}=0.75U_n+100$ for every n.
2) Consider, for every integer n, the sequence (V_n) such that $V_n = U_n - 400$.
   a- Prove that (V_n) is a geometric sequence whose common ratio and first term are to be determined.
   b- Calculate V_n in terms of n and deduce U_n in terms of n.
   c- Prove that the sequence (V_n) is decreasing and deduce that (U_n) is decreasing.
3) Calculate the limit of the sequence (U_n).
4) Can the number of members who leave the club in one year be less than 80?
5) Calculate the total number of members who left the club till February 2010.
   (Give the answer to the nearest unit).

IV- (5 points)

A-

Let g be the function defined, on $]0;+\infty[$, by $g(x)=-x^2-2+2\ln x$.

1) a- Calculate $\lim\limits_{x\to+\infty} g(x)$.
   b- Calculate $\lim\limits_{x\to 0} g(x)$.
2) a- Calculate $g'(x)$ and set up the table of variations of g.
   b- Deduce the sign of $g(x)$ on $]0;+\infty[$.

B-

Let f be the function defined, on $]0;+\infty[$, by $f(x)=-x+1-2\frac{\ln x}{x}$ and let (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Calculate $\lim\limits_{x\to+\infty} f(x)$ and show that the line (d) of equation $y=-x+1$ is asymptote to (C).
   b- Study, according to the values of x, the relative positions of (C) and (d).
   c- Calculate $\lim\limits_{x\to 0} f(x)$ and deduce an asymptote to (C).
2) Verify that $f'(x) = \frac{g(x)}{x^2}$ and set up the table of variations of f.
3) Determine the coordinates of point E on (C) at which the tangent (D) to (C) is parallel to the asymptote (d).
4) Draw (d), (D) and (C).
5) Calculate $\int_1^e \frac{\ln x}{x}dx$ and deduce the area of the region bounded by the curve (C), the asymptote (d) and the two lines with equations $x=1$ and $x=e$.

1- (2 points)

fu the following table only one of the proposed answers, to each question, is correct. Write down the number of each question and give, with justification, the corresponding answer.

No	Questions	Answers			
		a	b	c	d
1	The solution set of the inequality $\ln(x^2+4x+5)>0$ is:	IR	]0; + oo[	JR.- {-2}	]-2;+oo[.
2	$c_9^{s} - c_8^{s} =$	1	C:	d_7^{5}	c_9^{6}
3	The average of the marks of the students on a math exam is 9. If the mark of every student is increased by 10%, then the average of the marks becomes:	10	9.5	9.9	9.1
4	lim ln(e2x +2) / 2x, x--t+oo	1	0	2	+oo

II- (5 points)

In the year 2010, a private television channel had 4000 subscribers. Every year the chanilel cancels 10 % of the subscriptions and receives 500 new subscribers.

Designate by u.the number of members in the year (2010 + n) ; So $u_0 = 4000$.

1) Justify that for every natural integer n we have: $u_{n+1} = 0.9u_n + 500$.
2) Consider the sequence (v_n) defined for every natural integer n by: $v_n = 5000 - u_n$.
   a- Show that the sequence (v_n) is a geometric sequence whose common ratio and first term are to be determined.
   b- Calculate v_n in terms of n and deduce u_n in terms of n.
3) Specify the sense of variations of the sequence (v_n) and deduce that of the sequence (u_n).
4) Calculate $\lim_{n \to +\infty} u_n$.
5) Find the number of subscribers to this channel in 2015.
6) Starting from which year would the number of subscribers to this channel exceed the number of subscribers in2010 by 15%?
7) Can this channel reach 5100 subscribers? Justify the answer.

III- (5points)

During the holidays season, a supermarket management decided to offer gift coupons to every customer who participates in a game. For this, the management placed at the supermarket entrance an urn that contains three red balls each carrying the number 5 000, two white balls each carrying the number 10 000 and one black ball carrying the number -10 000.

The customer who decides to participate in this game draws simultaneously and randomly three balls from the urn.

1) Calculate the probability of each of the following events:
   A: « the three drawn balls have the same colour »
   B: « the three drawn balls are of three different colours »
   C: « among the three drawn balls, exactly two have the same colour ».

2) Every customer who participates in the game receives a gift coupon whose value, in LL, is the sum of the numbers carried by the three balls drawn.
   Let X be the random variable that is equal to the value of the gift coupon received by the customer.

   a- Show that the possible values of X are: 0 ; 5000 ; 10 000 ; 15 000 ; 20 000 ; 25 000 and determine the probability distribution of X.
   b- What is the probability that a customer gets a gift coupon that enables him to buy an article that costs 18 000 LL?
   c- Calculate the expected value E(X).
   d- Estimate the amount paid by the management during a week if every day 50 customers participate in the proposed game.

IV- (5points)

Let f be the function that is defined on IR by $f(x)=x-1-2e^{-\frac{1}{2}x}$.

(C) is the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Calculate $\lim\limits_{x\to+\infty} f(x)$.
   b- Show that the line (d) with equation $y=x-1$ is asymptote to (C) and determine the position of (C) relative to (d).
   c- Calculate $\lim\limits_{x\to-\infty} f(x)$ and give f (-2) in decimal form.
2) Calculate f '(x) and set up the table of variations of f.
3) Show that the equation $2e^{-\frac{1}{2}x}=x-1$ has a unique solution α and verify that $1.7<\alpha<1.9$.
4) Determine the coordinates of the point A on (C) at which the tangent (D) is parallel to the line with equation $y=2x$ and write an equation of (D).
5) Draw (d), (D) and (C).
6) Calculate the area of the region bounded by the curve (C), the asymptote (d) and the two lines with equations $x=0$ and $x=1$.
7) a- Show that the function f has on IR an inverse function g.
   b- Show that the equation f(x) = g(x) has no solution.

1- (3 points)

The table below shows the distribution of the monthly salaries of 40 employees (30 men and 10 women) in a company, in hundreds thousands LL.

Salaries	8	10	11.5	12	14	15	17.5
Frequency	8	4	x	10	y	7	2

The average monthly salary of the men is 1150 000 LL and the average monthly salary of the women is 1350 000 LL.

1) Show that the average monthly salary of all these employees is **1** 200 OOOLL.
2) Find the values of x and y.
3) The management of the company decides to increase the average monthly salary of the men by a certain ammmt so thathe average monthly salary of all the employees becomes 1260 OOOLL.

   Find the new average salary of the men in this company.

n- (4 points)

In order to motivate its clients, the management of a supermarket places 8 bills of 5 000 LL, **4** bills of 10 OOOLL and 2 bills of20 OOOLL in a box.

The client who purchases for more than 100 000 LL selects simultaneously and at random three bills of the box.

- If the selected bills have three different values, the client gains the three bills.
- If t\vo of the three selected bills have the same value and the third has a different value, the client gains the bill with the non-repeated value.
- If th_e three bills have the same value, the client gains nothing.

1) Determine the probability of each of the following events :

   A : «The client selects three 5 000 LL bills » ;

   B : « The client selects three 10 000 LL bills » ;

   C: «The client selects three bills of three different values».

2) Denote by X the random variable equal to the sum gained by a client that participates in this draw.

   a- Justify that the 5 possible values ofX are: 0; 5000; 10 000; 20 000 and 35 000.

   b- Prove that the probability $P(X = 10\ 000) = \frac{29}{91}$.

   c- Determine the probability distribution of X.

   d- Estimate the sum paid by the management during a week knowing that 100 clients participate in the draw each day.

ID- (5 points)

Ziad installs a water tank of capacity 20 000 liters in his house. The content of this tank is used as a supplement to the quantity of water that usually supplies the house.

Ziad fills the tank completely on the 31st of December 2010 and controls the water usage as follows :

- He uses, each month, 10% of the water content in the tank.
- He adds, in the last day of the month, 1000 liters of water to the content of the tank. Denote by C_n the content of the tank at the end of the nth month (the content is expressed in liters). Thus, C_0= 20 000.

1) Show that $C_{n+1} = 0.9\, C_n + 1000$, for all natural numbers n.

2) Consider the sequence (U_n) defined by $U_n = C_n - 10\ 000$.

a- Show that (U_n) is a geometric sequence whose common ratio and first term are to be determined.

b- Express U_n and C_n in terms of n.

c- Determine the sense of variations of the sequence (C_n)-

d- Calculate the limit of the sequence (C_n)-

3) Justify that the content of the tank always surpasses half its capacity.

4)a- Find the content of the tank on the 31st of December 2011. b- Calculate the quantity of water used by Ziad in 2011.

IV- (8 points)

A- Consider the function g defined over IR by g (x) = -x -2+2ex_

1) Calculate lim g(x) and lim g(x) .
X-t---= X-H-oo

2) Calculate g '(x) and set up the table of variations of g.

3) Verify that the equation g (x) = 0 has two roots 0 and a such that -1.6 < a < -1.5.

(In what follows, let a= -L55)

4) Determine, according to the values of x, the sign of g (x).

B- Consider the function f defined over IR by f (x) =ex. (ex-x-1) and denote by{C) its representative curve in an orthonormal system (O ; i,)) .

1) a- Calculate lim f(x) and deduce an asymptote to (C).
x-7-.oo

b- Calculate lim f(x) and fmd f(1.5) in decimal form.
X-t+oo

2) a- Calculate f' (x) and verify that f' (x) =ex g (x). b- Set up the table of variations of f.

3) Draw (C).

4) Calculate Jxexdx and deduce the area of the region bounded by (C), the x-axis, the y-axis and the straight line with equation x =1.