First exercise (7,5 points): Period of a simple pendulum – Forced oscillations

A simple pendulum is formed of a mass less inextensible string, supporting at one of its extremities a point mass (of very small dimensions), the other extremity of the string is fixed to a horizontal axis (Δ).
We designate by ℓ and m the length of the string and its mass respectively.

The pendulum oscillates in the vertical plane. We designate by θ the angle formed between the pendulum and descending vertical passing through (Δ) as shown in figure 1.

In this exercise we intend to study the proper period of a simple pendulum, for small amplitudes, with respect to initial conditions, its length, its mass and g.

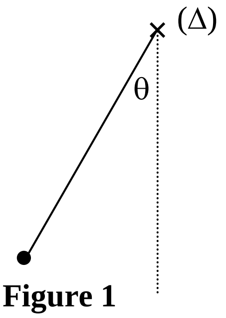

Figure 1

A – Influence of the initial conditions

Web ring a simple pendulum of length $\ell = 2\,\text{m}$, carrying a mass m = 100 g. We use this pendulum in two experiments with different launching initial conditions and we represent in the figures figures 2.a, 2.b the curves which show the variation of θ as a function of time.

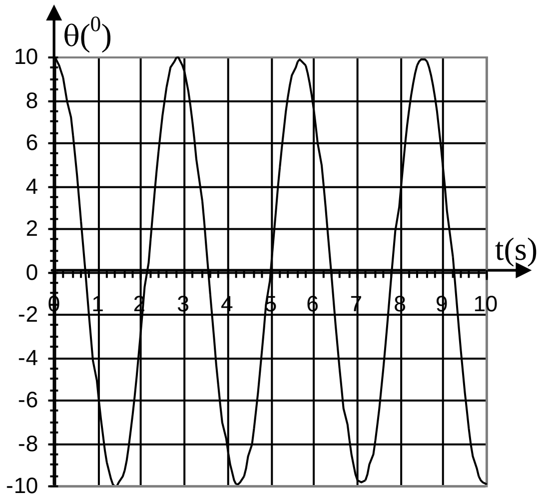

Figure 2.a

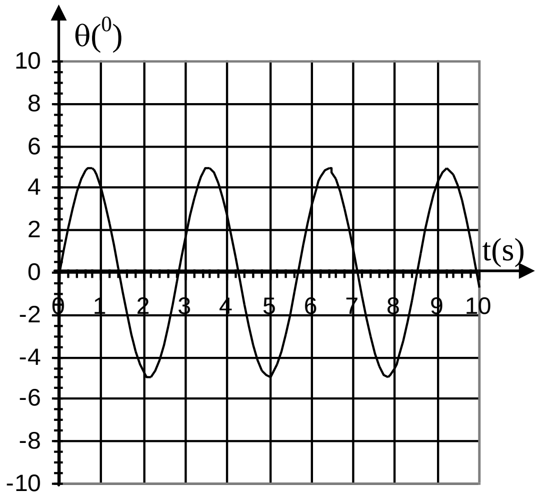

Figure 2.b

1) What is the nature of the oscillations in the two experiments? What do we name the period corresponding to each oscillation ?
2) Are the initial conditions (θ_0 and V_0) in the two preceding experiments identical?
3) Calculate the periods presented in each experiment.
4) Does the period depend on initial conditions? Justify.

B – Influence of mass

We consider two pendulums of the same length $\ell = 2\,\text{m}$ of different masses m = 100 g and m' = 150 g. the fixation points of the pendulums are taken to be on the same axis but separated by a certain distance. We move the two pendulums in the same direction and with the same angle $\theta_m = 15^0$ and we launch them without initial speeds at the same instant. We note that, during their oscillations, leave together and get back together to the point they were launched from.

Give a conclusion concerning the influence of mass on the proper period of oscillations.

C – Influence of the length ℓ

Web ring four pendulums of different lengths and we measure the proper period T_0 corresponding to each length. The results are resumed in the table below.

ℓ (m)	0,4	1	1,8	2
T_0 (s)	1,27	2,0	2,7	2,84

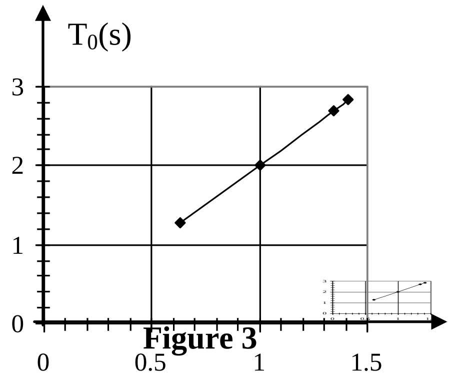

Figure 3

1) Is it important to know the mass and the initial conditions for each pendulum to come out with the preceding values of T_0? Why ?
2) Write briefly an experimental and practical method used to calculate each period in the table.
3) The graph giving the variation of T_0 as a function of $\sqrt{\ell}$ is given in figure 3. What is the relation between T_0 and ℓ ?

D – Influence of g and the expression of T_0.
We get a simple pendulum, of length $\ell = 2\,\text{m}$ carrying a small ball of iron of mass m = 100 g, an electromagnet functioning in a direct current.

The electromagnet placed just below the ball, exerts a magnetic force $\vec{F}$ practically vertical, descendante et constante when the pendulum oscillates with small amplitudes. The intensity of this magnetic force depends on the intensity of the current. The role of this force is to increase the attraction between the pendulum and earth which permits knowing the apparent gravitational acceleration $g' = \dfrac{mg + F}{m} = g + \dfrac{F}{m}$. We increase the value of F, we notice that the pendulum oscillates less and less faster.

La période propre d'un pendule simple oscillant en faible amplitude est l'une des expressions suivantes:

$T_0 = 2\pi\sqrt{\ell.g}$ (a) ; $T_0 = 2\pi\sqrt{\dfrac{\ell}{g}}$ (b) ; $T_0 = 2\pi\dfrac{\sqrt{\ell}}{g}$ (c) ; $T_0 = 2\pi\dfrac{\sqrt{\ell}}{mg}$ (d) ; $T_0 = 2\pi\sqrt{\dfrac{\ell.\theta_m}{g}}$ (e).

1) After studying the parts **A-**, **B-**, **C-** and **D-**, indicate, with justification, the incorrect expressions.
2) By a dimensional stuy of the unit deduce that (b) is the correct expression of the proper period of the pendulum.