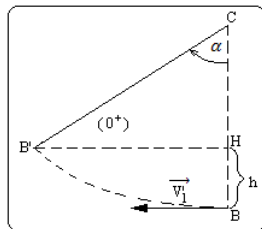


I- 1) No friction with air  $\Rightarrow$  M.E is conserved (0<sup>+</sup>)  
 $M.E_A = M.E_B$   
 $M.E_B = K.E_B + G.P.E_B$   
 $M.E_B = \frac{m_1 V_1^2}{2} = 1.25J$  (1/2)  
 $M.E_A = K.E_A + G.P.E_A \Rightarrow$   
 $1.25 = m_1 g L + \frac{m_A V_A^2}{2}$  (0<sup>+</sup>)  $\Rightarrow$   
 $V_A = 3m/s$  (1/2)

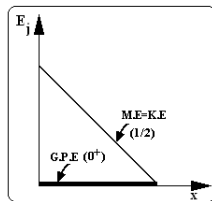
2) L.M is conserved  $\Rightarrow \vec{p}_f = \vec{p}_i$   
 $m_1 \vec{V}_1 + m_2 \vec{V}_2 = m_1 \vec{V}'_1 + m_2 \vec{V}'_2 \Rightarrow$   
 $m_1 \vec{V}'_1 + 0.4(2\vec{i}) = 0.1(5\vec{i}) + 0 \Rightarrow$   
 $\vec{V}'_1 = -3\vec{i}$  (1/2) ( $V'_1$  in m/s)  
 $K.E_f = \frac{m_1 V_1'^2}{2} + \frac{m_2 V_2'^2}{2} = 1.25J$   
 $K.E_i = \frac{m_1 V_1^2}{2} + \frac{m_2 V_2^2}{2} = 1.25J$   
 $K.E_f = K.E_i \Rightarrow$  Elastic collision (1)

3)  $\vec{V}'_1 = -3\vec{i} \Rightarrow$  Deviation to left (0<sup>+</sup>)  
 M.E is conserved  $M.E_B = M.E_{B'}$   
 $\frac{m_1 V_1'^2}{2} = m_1 g h \Rightarrow$   
 $h = 0.45m$  (0<sup>+</sup>)  
 $h = CB - CH =$   
 $h = L - L \cos \alpha$  (0<sup>+</sup>)  
 $0.45 = 0.8 - 0.8 \cos \alpha$   
 $\Rightarrow \alpha = 64^\circ$  (1/2)



4) a)  $\Delta M.E = W_{\vec{f}} \Rightarrow 0 - \frac{m_2 V_2'^2}{2} = -f \cdot d$  (0<sup>+</sup>)  
 $f = 0.16N$  (1/2)

b) G.P.E = 0 (0<sup>+</sup>)  
 $M.E - M.E_0 = -f \cdot x$   
 $M.E = -0.16x + 0.8$  (0<sup>+</sup>)  
 $K.E = M.E$  (0<sup>+</sup>)



$x = x_m \sin(\omega_0 t + \varphi)$   
 $\frac{1}{2} m v_0^2 = \frac{1}{2} K x_m^2 \Rightarrow x_m = 0.2m$  (1/2)  
 $V = \omega_0 x_m \cos(\omega_0 t + \varphi) \quad V_0 = \omega_0 x_m \cos(\varphi) > 0$   
 $t = 0, x = 0 \Rightarrow 0 = x_m \sin(\varphi) \Rightarrow$   
 $\sin \varphi = 0 \Rightarrow \varphi = 0 \text{ or } \varphi = \pi$   
 $\varphi = 0 \Rightarrow V_0 > 0$  accepted (0<sup>+</sup>)  
 $\varphi = \pi \Rightarrow V_0 < 0$  rejected (0<sup>+</sup>)  
 $\Rightarrow x = 0.2 \sin(10t)$  (0<sup>+</sup>) ( $x$  in m,  $t$  in s)

4) a)  $S = S_0 + \Delta S = S_0 + L \cdot x = S_0 + 0.25x$  (0<sup>+</sup>)  
 $\Phi = B \cdot S \cdot \cos(180^\circ) = -B(S_0 + 0.25x)$   
 (0<sup>+</sup>)  
 $\Phi = -0.4(S_0 + 0.25x)$  (0<sup>+</sup>)

b)  $e = -\frac{d\Phi}{dt} = BLx' = BLV$  (1/2)  
 $e_0 = B \cdot L \cdot V_0 = 0.2V$  (1/2)

c)  $I_0 = \frac{e_0}{R} = 0$  very large R (1/2)

d) when  $R = 5\Omega \Rightarrow I_0 = \frac{0.2}{5} = 0.04 \text{ A}$  (1/2)

The induced current  $I_0$  acts in such a way to oppose the cause producing it  $\Rightarrow$  The electromagnetic force  $\vec{F}_0$  is opposite in direction to  $\vec{V}_0$

In magnitude:  $F_0 = I_0 \cdot B \cdot L \cdot \sin(\widehat{I_0, \vec{B}})$   
 $F_0 = 4 \times 10^{-3} N$  (1/2)

The electromagnetic force  $\vec{F}_{em}$  acting on MN is opposite to  $\vec{V}$  in direction  $\Rightarrow$  it damps the oscillations

$\Rightarrow$  the oscillation of MN is damped.

In order to produce S.H.M, we must exert on MN a driving force  $\vec{F}$  opposite to  $\vec{F}_{em}$ . (0<sup>+</sup>)

The oscillation of MN is called driven oscillation. (0<sup>+</sup>)

II-1)  
 $M.E = \frac{1}{2} m V^2 + \frac{1}{2} K x^2 = K.E + E.P.E$  (0<sup>+</sup>)  
 2)  $\frac{d(M.E)}{dt} = 0 \Rightarrow x'' + \frac{K}{m} x = 0$  (0<sup>+</sup>)  
 $\Rightarrow x'' + \omega_0^2 x = 0 \quad \omega_0 = \sqrt{\frac{K}{m}}$  (0<sup>+</sup>),  
 $T = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{m}{K}}$  (0<sup>+</sup>)  
 3)  $\omega_0 = \sqrt{\frac{50}{0.5}} = 10 \text{ rad/s}$  (0<sup>+</sup>)

III- 1)  
 a)  $U_{m_1} = 2 \times 3 = 6V$  (0<sup>+</sup>),  $U_{m_2} = 2 \times 1$  (0<sup>+</sup>)  
 $U_{m_1} > U_{m_2} \Rightarrow$  curve  $C_1$  represents  $u_G$  (0<sup>+</sup>)

b)  $U_{m_2} = R \cdot I_m \Rightarrow$   
 $I_m = 0.1A$  (0<sup>+</sup>)

$\varphi = 2\pi \frac{d}{D} = \frac{\pi}{3} \text{ rd}$  (0<sup>+</sup>)

$u_G$  cuts t-axis before  $u_R$   
 $\Rightarrow u_G$  leads  $i$  (0<sup>+</sup>)

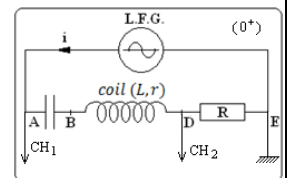
$T = 6 \times 2 = 12 \text{ ms} = 0.012 \text{ S}$  (0<sup>+</sup>)

$\omega = \frac{2\pi}{T} = \frac{500\pi}{3} \text{ rd/s}$  (0<sup>+</sup>)

$u = 6 \sin(\frac{500\pi}{3} t)$  (0<sup>+</sup>),  $i = 0.1 \sin(\frac{500\pi}{3} t - \frac{\pi}{3})$  (0<sup>+</sup>)

c)  $P_{av} = U \cdot I \cdot \cos(\varphi) = 0.15 \text{ W}$  (1/2)

$P_{av} = (R+r) I^2 \Rightarrow r = 10\Omega$  (1/2)



a) current resonance ( $0^+$ )  $LC\omega_0^2=1 \Rightarrow L=0.05H$  (1/2)

b)  $U_m = (R+r) I_m' \Rightarrow I_m' = 0.2A$  ( $0^+$ )  
 $i' = 0.2\sin(200\pi t)$  ( $0^+$ )

c)  $u_C = \frac{q}{C} = \frac{1}{C} \int i. dt = \frac{0.2}{50 \times 10^{-6}} \int \sin(200\pi t)$

$u_C = -6.28 \cos(200\pi t)$  ( $0^+$ )

$u_{coil} = ri + L \frac{di}{dt} = 2\sin(200\pi t) + 6.28\cos(200\pi t)$

$u_{coil} = U_{m(r,L)} \sin(\omega t + \varphi') \Rightarrow$

Calc.  $\tan\varphi' = \frac{L\omega}{r} = 3.14 \Rightarrow$

$\varphi' = 1.26\text{rd} = 72^\circ$  (1/2)

$U_{m(r,L)} = I_m' \sqrt{r^2 + (L\omega)^2}$

$u_{coil} = 6.6\sin(200\pi t + 1.26)$  ( $0^+$ )

IV-1)

a)  $a = \frac{mx+M(0)}{m+M} = \frac{mx}{m+M} = \frac{x}{4}$  ( $0^+$ )

b)  $I = I_{rod} + I_m = \frac{ML^2}{12} + mx^2 = \frac{1}{20} + 0.2x^2$  ( $0^+$ )

c) i)  $K.E + G.P.E = M.E = \frac{I\theta'^2}{2} - mgh$  ( $0^+$ ),

$h = x \cos \theta$

$M.E = \frac{I\theta'^2}{2} - 0.2(10)(x \cos \theta) \Rightarrow$

$M.E = \left(\frac{1}{40} + 0.1x^2\right) \theta'^2 - 2x \cos \theta$

$M.E = \left(\frac{1}{40} + 0.1x^2\right) \theta'^2 - 2x + x\theta^2$  (1/2)

ii) No friction  $\Rightarrow$  M.E is conserved  $\Rightarrow$

$\frac{dM.E}{dt} = 0$  ( $0^+$ ),  $x = \text{constant} \Rightarrow$

$\theta'' + \left(\frac{x}{\frac{1}{40} + 0.1x^2}\right) \theta = 0$  ( $0^+$ )

Or  $\theta'' + \frac{(M+m).g.a}{I} \theta = 0$

Similar to  $\theta'' + \omega_0^2 \theta = 0$

$\Rightarrow \omega_0 = \sqrt{\frac{x}{\frac{1}{40} + 0.1x^2}}$  ( $0^+$ )

$T_0 = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{\frac{1}{40} + 0.1x^2}{x}} = 2\pi \sqrt{\frac{1}{40} + \frac{x}{10}}$  (1/2)

iii)  $T_0$  is min  $\Rightarrow \frac{1}{40x} + \frac{x}{10}$  is min  $\Rightarrow$

$\frac{1}{40x} = \frac{x}{10}$  (using derivative or other method)

$\Rightarrow x = 0.5\text{m}$  (1/2)

$T_{0(\text{min})} = 2\text{S}$  ( $0^+$ ),  $\pi^2 = 10$

iv)  $2.5 = 2\pi \sqrt{\frac{1}{40x} + \frac{x}{10}} \Rightarrow$

$4x^2 - 6.25x + 1 = 0$  ( $0^+$ )

$\Rightarrow x = 0.16\text{m} < \frac{1}{2}$  (accepted) (1/2)

$x = 1.38\text{m}$  (rejected) ( $0^+$ )

a)  $P.E = T.E_{P.E} + G.P.E = \frac{1}{20} C \theta^2 - mg \frac{L}{2} \cos \theta$  (1/2)

$P.E = \frac{1}{2} C \theta^2 - \cos \theta = \frac{1}{2} C \theta^2 + \frac{1}{2} \theta^2 - 1$

$P.E = \frac{1}{2} (C + 1) \theta^2 - 1$  ( $0^+$ )

b)  $M.E = K.E + P.E = \frac{1}{2} I \cdot \theta'^2 + \frac{1}{2} (C + 1) \theta^2 - 1$

$M.E = \frac{1}{20} \theta'^2 + \frac{1}{2} (1 + C) \theta^2 - 1$  (1/2)

No friction  $\Rightarrow$  M.E is conserved  $\Rightarrow \frac{dM.E}{dt} = 0$

$\Rightarrow \theta'' + 10(1 + C)\theta = 0$  (1/2)

Similar to  $\theta'' + \omega_0^2 \theta = 0$

c)  $\Rightarrow \omega_0 = \sqrt{10(1 + C)}$  ( $0^+$ )

$\Rightarrow T_0 = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{1}{10(1 + C)}}$  (1/2)

d)  $1.8 = 2\pi \sqrt{\frac{1}{10(1 + C)}} \Rightarrow C = 0.23 \text{ S.I}$  (1/2)

