3rd secondary class(LS)

Mathematics (Term 1 test)

Exercise one:(2 points)

Choose the correct answer with justification:

1) Given $z = sin50° + isin40°$, the exponential form of $u = z\bar{z} + i$ is:

   a)-$\sqrt{2}e^{i\frac{5\pi}{4}}$ b)$\sqrt{2}e^{i\frac{\pi}{4}}$ c)$1 + i$ d)non of these.

2) $T = \frac{(1+i)^9}{(1-i)^8}$. T is equal to:

   a)$1 - i$ b)$1 + i$ c)$-1 + i$ d)$-1 - i$

Exercise two:(4 points)

The four parts of this exercise are independent.

1) $f(x) = x - \ln(1 + e^x)$. Determine $\lim_{x \to +\infty} f(x)$.
2) Solve the equation: $e^{2x} - e^{x + ln2} = 3$.
3) Solve the inequality: $\ln(x^2 - x) \leq ln2 + ln3$.
4) a- Verify that for all , $\frac{e^{2x}-1}{e^{2x}+1} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

   b-Deduce $\int \frac{e^{2x}-1}{e^{2x}+1} dx$.

Exercise three:(2 points)

In the space of an orthonormal reference $\left(O; \vec{i}, \vec{j}, \vec{k}\right)$, consider the points

A(1 , 0 , 0), B(1 , 1 , 1), C(2 , 3 , 0), and D(2 , 0 , 3).

1) Verify that ABCD is a tetrahedron whose volume is to be calculated.
2) Prove that the two straight-lines (AB) and (CD) are orthogonal.
3) Let H$\left(\frac{17}{11}, \frac{9}{11}, \frac{9}{11}\right)$ and I be the midpoint of [CD]. Prove that A, H, and I are collinear.

Exercise three:(4 points)

In the complex plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points A,E ,M and M' so that $z_A = 2$, $z_E = 4$, $z_M = z$, $z_{M'} = z'$ with $z' = \frac{-4}{z-2}$.

1) a-Solve the equation $z' = z$.

   b-Denote by z_B and z_C the previous solutions (B lies in quadrant IV). Show that the exponential forms of z_B and z_C are $2e^{-i\frac{\pi}{3}}$ and $2e^{i\frac{\pi}{3}}$ *respectively then* Plot all the given points.

   c-Find the exponential form of $\frac{z_E - z_B}{z_E - z_C}$ and deduce the nature of triangle EBC.

2) a-Calculate $z' - 2$ in terms of z and deduce that $AM' = \frac{2OM}{AM}$

   b-Find the locus of M' when M moves on line (BC)

3) a-Verify that $\left(\vec{u}, \overrightarrow{AM'}\right) = \pi + \left(\overrightarrow{AM}, \overrightarrow{OM}\right) + 2k\pi$

   b-Find the locus of M' when M moves on a circle of diameter [AO]

<u>Exercise four:(8 points)</u>

Let f be a function defined on IR by $f(x) = e^{-2x} - 2e^{-x} + 1$ and (C) be its representative curve in an orthonormal system $(O; \vec{\imath}, \vec{\jmath})$. (unit=2cm)

1) Calculate $\lim_{x\to+\infty} f(x)$ then deduce an asymptote $(d)$to (C).
2) Calculate $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to-\infty} \frac{f(x)}{x}$
3) Determine $f'(x)$ and set the table of variations of f.
4) Prove that (C)has an inflection point I whose coordinates are to be determined
5) Determine the coordinates of the intersection point of $(C)$and (d).
6) Draw $(d)$and (C).
7) Let g be the function given by $g(x) = \ln(f(x))$and let (G)be its representative curve.
   a- Justify that the domain of definition of g is $]-\infty, 0[\cup]0, +\infty[$ and set up its table of variations
   b- Prove that the line (D) of equation $y = -2x$ is an asymptote to (G).
   c- Solve the equations $g(x) = 0$ and $g(x) = -2x$.
   d- Draw $(D)$and (G) in a new system of axes.

Life science section (Distribution of marks over 20)

Exercise one:

$u = \|z\|^2 + i = 1 + i \to u = \sqrt{2}e^{i\frac{\pi}{4}}$	1 point
$\|T\| = \sqrt{2} \quad and\, \arg(T) = \frac{\pi}{4} + 2k\pi \to T = 1 + i$	1 point

Exercise two:

1) $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} ln\left(\frac{e^x}{1+e^x}\right) = \lim\limits_{x\downarrow+\infty} ln\left(\frac{e^x}{e^x}\right) = ln1 = 0$	1 point
2) $x = ln3$	1 point
3) $x \in [-2,0[\cup]1,3]$	1 point
4) a- $e^{-x} = \frac{1}{e^x} \to \frac{e^{2x}-1}{e^{2x}+1} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. b- $\int \frac{e^{2x}-1}{e^{2x}+1} dx = ln(e^x + e^{-x}) + k$	0.5 pt. 0.5 pt.

Exercise three:

$\overrightarrow{AB}.\left(\overrightarrow{AC} \times \overrightarrow{AD}\right) = -6 \neq 0 \to ABCD\ is\ a\ tetrahedron$ $V = \frac{1}{6} \times 6 = 1\ unit\ cube$	0.75pt.
$\overrightarrow{AB}.\overrightarrow{CD} = 0 \to \overrightarrow{AB} and \overrightarrow{CD}\ are\ orthogonal$	0.5 pt.
$I\left(2, \frac{3}{2}, \frac{3}{2}\right)\ AND\ \overrightarrow{IA} \times \overrightarrow{IH} = \vec{0} \to I, A\ AND\ H\ are\ collinear$	0.75 pt.

Exercise four:

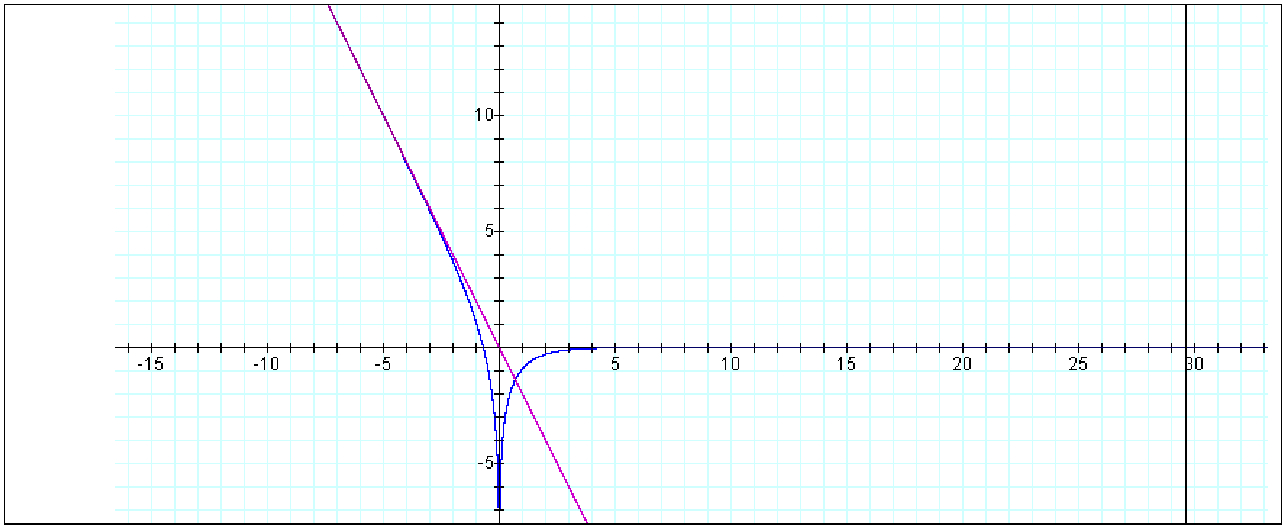
1) a- $z_1 = 1 + i\sqrt{3}\ and\ z_2 = 1 - i\sqrt{3}$ b- $z_C = 2e^{i\frac{\pi}{3}}\ and\ z_B = 2e^{-i\frac{\pi}{3}}$ c- $\frac{z_E - B}{z_E - z_C} = e^{i\frac{\pi}{3}}$ The nature of triangle EBC is equilateral triangle.	0.5 0.5 1
2) a- $\|z' - 2\| = \left\|\frac{-2z}{z-2}\right\| \to AM' = \frac{2OM}{AM}$	0.5

b- (BC) *perpendicular bisector of* $[\![OA]\!] \rightarrow AM' = 2$ $\rightarrow M'$ *moves on a circle center* A *radius* $2.$	0.5
3) a- $\arg(z'-z) = \arg(-2z) - \arg(z-2) + 2k\pi$ b- M' *moves on a straight line perpendicular to* $x-axis$ *passing through* A	0.5 0.5

1) $\lim_{x\to+\infty} f(x) = 1$, $y = 1$ $H.A$	0.5
2) $\lim_{x\to-\infty} f(x) = +\infty$, $\lim_{x\to-\infty} \frac{f(x)}{x} = -\infty$	0.5
3) $f'(x) = 2e^{-x}(-e^{-x}+1)$	0.5 0.75
4) $f''(x) = 2e^{-x}(2e^{-x}-1)$, $f''(x) = 0$ *iff* $x = ln2$ *and* f'' *changes its sign at* $ln2$ *, so* (C) *has an inflection point* $I(ln2, 0.25)$	0.75
5) $(C) \cap (d) : (-ln2, 1)$	0.5
6) 	1
7)a- $f(x) = 0$ *when* $x = 0$ *s0* $f(x) > 0$ *iff* $x \neq 0$ *so* $D.g :]-\infty, 0[\cup]0, +\infty[$ b- $\lim_{x\to-\infty} g(x) + 2x = 0$ c- $g(x) = 0 \rightarrow x = -ln2$, $g(x) = -2x \rightarrow x = ln2$ d- Variations of g and its graph.	0.5 0.5 1 1.5

Variation table for 3):

x	$-\infty$	0	$+\infty$
$f'(x)$	_	0	+
$f(x)$	decreasing		increasing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