I- (4 points)

In the complex plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points A, B, D, M and M′ of respective affixes –3, –1 + i, –4 + 2i, z, and z′ such that $z' = \dfrac{z+1-i}{z+3}$ ($z \neq -3$).

1) Write z′ in exponential form when z = –1 + 2i.

2)
   - **a-** Give a geometric interpretation of | z′ |, then deduce the set (L) of points M when | z′ | = 1.
   - **b-** Verify that I, the midpoint of [BD], belongs to (L).

3)
   - **a-** Prove that | z′ – 1 | × | z + 3 | is a constant to be determined.
   - **b-** Let E be a point of affix 1 and (C) be the circle of center E and radius $\sqrt{5}$. Prove that, if M′ moves on (C), then M moves on a circle whose center and radius are to be determined.

4)
   - **a-** Prove that $\dfrac{z_D - z_A}{z_B - z_A}$ is pure imaginary.
   - **b-** Deduce the nature of triangle ABD.

II- (4 points)

Given, in the space of an orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, the point A(2 ; 1 ; 5) and the two straight lines (d) and (d') defined by: (d) $\begin{cases} x = 2m+4 \\ y = 2m+1 \\ z = -3m-5 \end{cases}$ and (d′) $\begin{cases} x = t+2 \\ y = 2t-1 \\ z = -2t+6 \end{cases}$, where m and t are real parameters.

1) Prove that (d) and (d′) are skew.
2) Show that the plane (P) determined by A and (d′) has a cartesian equation: ax + y + cz – 15 = 0, where a and c are real numbers to be determined.
3) Let M be a variable point of (d).
   - **a-** Calculate the distance from M to (P), then deduce the relative position of (d) with respect to (P).
   - **b-** (d) and A determine a plane (Q). Without finding an equation of (Q), find a system of parametric equations of the line of intersection of (P) and (Q).
   - **c-** Let f(m) = AM^2. Calculate, in terms of m, f(m) and deduce the coordinates of the orthogonal projection of A on (d).

III- (4 points)

U_1 and U_2 are two given urns such that:

U_1 contains 10 balls: 6 red and 4 yellow.

U_2 contains 10 balls: 5 red, 4 black and 1 green.

C is a given fake coin such that the probability of having a head is three times more than that of having a tail.

We throw coin C.

- If we get a tail, we pick up, randomly and **simultaneously**, two balls from urn U_1
- If we get a head, we select at random two balls from U_2 **one after the other with replacement**.

Consider the following events:

U_1: "The chosen urn is U_1."

U_2: "The chosen urn is U_2."

R: "The chosen balls are red."

1) Show that $P(U_2) = \frac{3}{4}$ and $P(U_1) = \frac{1}{4}$.
2) Calculate $P(R/U_1)$, $P(R \cap U_1)$, and $P(R \cap U_2)$. Deduce that $P(R) = \frac{13}{48}$.
3) The two selected balls are red. Calculate the probability that they come from U_1.
4) Let X be the random variable that designates the number of red balls obtained. Determine the probability distribution of X.

IV- (8 points)

Part A

Consider the differential equation (E): $y'' - 3y' + 2y = 2xe^x$. Let $y = z - x^2e^x - 2xe^x$.

1) Form the differential equation (F) satisfied by z, then solve it.
2) Deduce the general solution of (E), and the particular solution whose representative curve (C), in an orthonormal system $(O;\vec{i},\vec{j})$, admits at the point A(0 , 2) a horizontal tangent.

Part B

Let f be a function defined, on $\mathbb{R}$, by: $f(x) = (-x^2 - 2x + 2)e^x$. Let (C) be the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

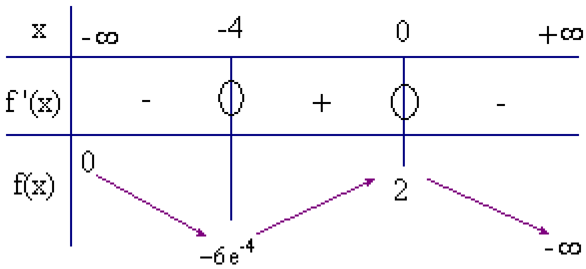
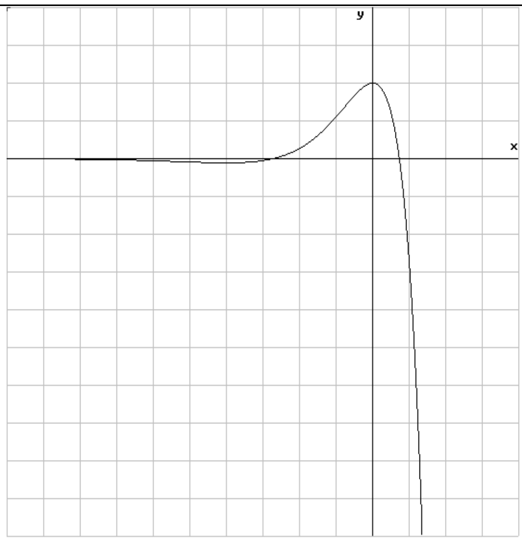
1) Find the limits of f at $-\infty$ and at $+\infty$.
2) Prove that $f'(x) = -x(x+4)e^x$, then construct the table of variations of f.
3) Prove that the equation f(x) = 0 has two roots α and β. Verify that $-2.8 < \alpha < -2.7$ and that $0.7 < \beta < 0.8$.
4) Admit that $f''(x) = -(x^2 + 6x + 4)e^x$. Verify that (C) has two inflection points whose coordinates are to be determined.
5) Draw (C).
6) Calculate the area of the domain bounded by (C), the abscissa axis, the ordinate axis, and the straight-line $x = -1$.

Answer Key

Question I			Mark
1)		$z = \frac{\sqrt{2}}{8} e^{\frac{\pi}{4}i}$	**0.5**
2)	**a-**	• $\lvert z' \rvert = \frac{BM}{AM}$ or $\lvert z' \rvert = OM'$ • (L) is the perpendicular bisector of [AB]	**0.25** **0.75**
	b-	$z_I = -\frac{5}{2} + \frac{3}{2}i$; $\left\lvert \frac{z_I + 1 - i}{z_I + 3} \right\rvert = 1$, then $I \in (L)$	**0.5**
3)	**a-**	$\lvert z' - 1 \rvert \times \lvert z + 3 \rvert = \lvert -2 - i \rvert = \sqrt{5}$	**0.5**
	b-	$EM' \times AM = \sqrt{5}$; AM = 1; M moves on the circle of center A and radius 1	**0.75**
4)	**a-**	$\frac{z_D - z_A}{z_B - z_A} = i$; then $\frac{z_D - z_A}{z_B - z_A}$ is pure imaginary	**0.25**
	b-	ABD isosceles (AD = AB) and right at A	**0.5**

Question II			Mark
1)		$\overrightarrow{BC}.\left(\vec{u}_d \wedge \vec{u}_{d'}\right) = 16 \neq 0$; then (d) and (d') are skew	**1**
2)		(P): $2x + y + 2z - 15 = 0$	**0.5**
3)	**a-**	• $d(M, (P)) = \frac{15}{3}u$ • (d) / / (P)	**0.5** **0.5**
	b-	(L): $\begin{cases} x = 2k + 2 \\ y = 2k + 1 \\ z = -3k + 5 \end{cases}$ (k is a real number)	**0.5**
	c-	• $f(m) = AM^2 = 17m^2 + 68m + 104$ • (0 , –3 , 1)	**0.5** **0.5**

Question III		Mark
1)	$P(U_1) + P(U_2) = 1$ and $P(U_2) = 3P(U_1)$; then $P(U_2) = \frac{3}{4}$ and $P(U_1) = \frac{1}{4}$	**0.5**
2)	$P(R/U_1) = \frac{1}{3}$; $P(R \cap U_1) = \frac{1}{12}$; $P(R \cap U_2) = \frac{3}{16}$; $P(R) = \frac{13}{48}$	**0.25** **0.5** **0.5** **0.5**
3)	$P(U_1/R) = \frac{4}{13}$	**0.5**
4)	$X_\Omega = \{0, 1, 2\}$; $P(X = 0) = \frac{53}{240}$; $P(X = 1) = \frac{61}{120}$; $P(X = 2) = \frac{13}{48}$	**0.25** **0.5** **0.5**

	Question IV	Mark
Part A		
1)	• (F): $z'' - 3z' + 2z = 0$ • $z = C_1e^x + C_2e^{2x}$	**0.75** **0.5**
2)	• GS of (E): $y = C_1e^x + C_2e^{2x} - x^2e^x - 2xe^x$ • $y(0) = 2$, then $C_1 + C_2 = 2$; $y'(0) = 0$, then $C_1 + 2C_2 = 2$; $C_1 = 2$ and $C_2 = 0$; $y = \left(-x^2 - 2x + 2\right)e^x$	**0.25** **0.5**
Part B		
1)	0; $-\infty$	**0.25** **0.25**
2)	• $f'(x) = -x(x+4)e^x$ 	**0.5** **1**
3)	On $]-4, 0[$: f is continuous and strictly increasing from -0.11 to $+2$, then $f(x) = 0$ has a unique root α. But $f(-2.8) \times f(-2.7) < 0$, then $-2.8 < \alpha < -2.7$. Similarly, $0.7 < \beta < 0.8$	**0.5** **0.5**
4)	When $x = -5.2$: $f''(x) = 0$ and $f''(x)$ changes signs ($-$ to $+$), then $I_1(-5.2, -0.1)$ When $x = -0.8$: $f''(x) = 0$ and $f''(x)$ changes signs ($+$ to $-$), then $I_2(-0.8, 1.3)$	**0.5** **0.5**
5)	 **6)** Area $= \int_{-1}^{0} f(x)ds = 2 - e^{-1} = 1.63\ u^2$	**1** **1**