I- (4 points)

OABCDEFG is a cube of edge 1. The space is referred to an orthonormal system $\left(O;\overrightarrow{OA},\overrightarrow{OC},\overrightarrow{OD}\right)$. Let a be a strictly positive real number. M, N, and P are three points such that M(a , 0 , 0), N(0 , a , 0), and P(1 , 1 , a).

1) Prove that the cartesian equation of the plane (DMN) is: $x + y + az = a$.

2)

a- Prove that $\overrightarrow{OP}$ is a normal vector of plane (DMN), then give a parametric representation of the straight-line (OP).

b- The straight-line (OP) cuts the plane (DMN) at H. Express, in terms of a, the real number t such that $\overrightarrow{OH} = t\overrightarrow{OP}$.

3)

a- Calculate, in terms of a, the distance from P to plane (DMN).

b- Justify that the triangle DMN is isosceles. Calculate, in terms of a, the area of triangle DMN.

c- Deduce the volume of the tetrahedron PDMN.

II- (4 points)

In the complex plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points A, B, M and M′ of respective affixes 2i, 1 + i, z, and z′ such that $z' = \dfrac{z-2i}{z}$ $(z \neq 0)$.

1) Let (C) be the circle of center O and radius 1.

a- Give a geometric interpretation of $|z'|$, then deduce the set (L) of points M when M′ moves on the circle (C). Verify that B belongs to (L).

b- Give a geometric interpretation of $\arg(z')$, then deduce the set of points M when z′ is real.

2) Let $z = e^{\frac{\pi}{3}i}$. E and F are two points of affixes z^n and z^{n+1} respectively, where n is a natural number.

a- Can you find the values of n so that z^n is pure imaginary? Justify your response.

b- Show that the triangle OEF is equilateral.

III- (4 points)

In a class of 30 students, there are two clubs formed. The first club is the photo club that contains 10 members and the second club is the theater club that contains 6 members. There are two students in the class who are members in the two clubs together.

1) We choose, randomly, one student from the class. Let P be the event: “The chosen student belongs to the photo club” and T be the event: “The chosen student belongs to the theater club”. Are P and T independent? Justify.

2) In a session of the photo club, the 10 members are present. One student is chosen randomly. This student has to take a picture for a second student in the club.

a- Let T_1 be the event: “The first chosen student belongs to the theater club”.
Calculate $P(T_1)$.

b- Let T_2 be the event: “The pictured student belongs to the theater club”.
Calculate $P(T_2/T_1)$, then $P(T_2/\overline{T_1})$. Deduce $P(T_2 \cap T_1)$ and $P(T_2 \cap \overline{T_1})$.

c- Prove that the probability that the pictured student belongs to the theater club is 0.2.

IV- (8 points)

Let f be a given function defined, on $\mathbb{R}$, by: $f(x) = x - \dfrac{e^x}{e^x + 1}$. Let (C) be the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Calculate the limits of f at $-\infty$ and at $+\infty$.
2) Let (D) and (L) be the two straight-lines of respective equations y = x and y = x – 1. Show that (D) is an asymptote to (C) at $-\infty$ and that (L) is an asymptote to (C) at $+\infty$.
3) Let $E\left(0, -\dfrac{1}{2}\right)$. Show that E is a center of symmetry of (C).
4)
   a- Calculate $f'(x)$ and deduce that $f'(x) > 0$.
   b- Set up the table of variations of f.
5)
   a- Write the equation of (T), the tangent to (C) at point E.
   b- Show that E belongs to (C) and (T), then study the relative positions of (C) and (T).
6) Prove that E is a point of inflection of (C).
7) Prove that the equation f(x) = 0 has a unique root α and verify that $0.6 < \alpha < 0.7$.
8) Draw (T) and (C).
9) The function f admits an inverse function f^{-1}.
   a- Draw (C′), the representative curve of f^{-1} in the same previous system.
   b- Let (T′) be the tangent to (C′) at the point $E'\left(-\dfrac{1}{2}, 0\right)$. Prove, without drawing (T′), that (T) and (T′) intersect at one point F whose coordinates are to be determined.

Answer Key

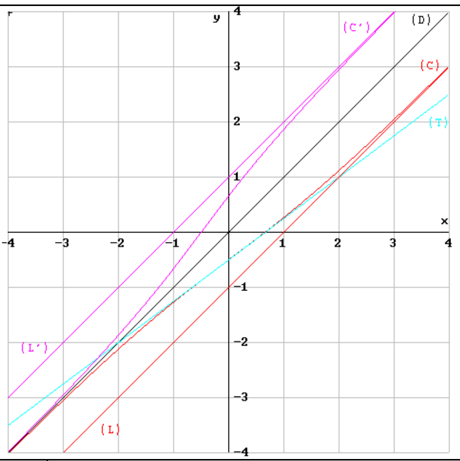
Question I			Mark
1)	D(0 , 0 , 1); D, M, and N belong to x + y + az = a		0.5
2)	a-	$\overrightarrow{OP}(1,1,a) = \overrightarrow{N}(1,1,a)$; (OP): $\begin{cases} x = k \\ y = k \\ z = ak \end{cases}$	0.25 0.5
	b-	{H} = (OP) ∩ (DMN); $H\left(\frac{a}{2+a^2};\frac{a}{2+a^2};\frac{a^2}{2+a^2}\right)$; t = $\frac{a}{2+a^2}$	0.75
3)	a-	d(P , (DMN)) = $\frac{\lvert a^2-a+2\rvert}{\sqrt{2+a^2}} = \frac{a^2-a+2}{\sqrt{2+a^2}}$ u	0.5
	b-	DM = DN = $\sqrt{a^2+2}$, then DMN isosceles. Area = $\frac{a\sqrt{2+a^2}}{2}$ units of area	0.5 0.5
	c-	Volume = d(P, (DMN)) × Area DMN = $\frac{a^3-a^2+2a}{2}$ units of volume	0.5

Question II			Mark
1)	a-	• $\lvert z' \rvert = \frac{AM}{OM}$. • OM′ = 1; $\lvert z' \rvert$ = 1; AM = OM; M moves on the perpendicular bisector of [OA]. • $\left\lvert\frac{z_B-2i}{z_B}\right\rvert = \left\lvert\frac{1-i}{1+i}\right\rvert$ = 1; B ∈ (L).	0.25 0.75 0.5
	b-	• arg(z′) = $(\overrightarrow{OM},\overrightarrow{AM})$ (2π). • arg(z′) = kπ; $(\overrightarrow{OM},\overrightarrow{AM})$ = kπ; O, A, and M are collinear; M moves on the straight-line (OA) deprived of O	0.25 0.75
2)	a-	arg(z^n) = $\frac{(2k+1)\pi}{2}$ (k ∈ Z); $\left(\frac{n\pi}{3}\right) = \frac{(2k+1)\pi}{2}$; n = $\frac{6k+3}{2}$ which is never natural; n cannot be found	0.5
	b-	OE = $\lvert z^n \rvert$ = 1; OF = $\lvert z^{n+1} \rvert$ = 1; $(\overrightarrow{OE},\overrightarrow{OF})$ = arg$\left(\frac{z^{n+1}}{z^n}\right)$ = arg(z) = $\frac{\pi}{3}$. OEF is equilateral being isosceles with one 60° angle	1

Question III			Mark
1)	P(P ∩ T) = $\frac{2}{30} = \frac{1}{15}$; P(P) = $\frac{10}{30} = \frac{1}{3}$; P(T) = $\frac{6}{30} = \frac{1}{5}$; P(P) × P(T) = P(P ∩ T), then P and T are independent		1
2)	a-	$P(T_1) = \frac{2}{10} = \frac{1}{5}$	0.5
	b-	$P(T_2/T_1) = \frac{1}{9}$; $P(T_2/\overline{T_1}) = \frac{2}{9}$; $P(T_2 \cap T_1) = \frac{1}{45}$; $P(T_2 \cap \overline{T_1}) = \frac{8}{45}$	0.5 each

	c-	$P(T_2) = P(T_2 \cap T_1) + P(T_2 \cap \overline{T_1}) = 0.2$	**0.5**

Question IV			**Mark**
1)		• $\lim\limits_{x \to -\infty} f(x) = -\infty$. • $\lim\limits_{x \to +\infty} f(x) = +\infty$.	**0.25** **0.25**
2)		• $\lim\limits_{x \to -\infty} \left(f(x) - y_{(D)}\right) = 0$. • $\lim\limits_{x \to +\infty} \left(f(x) - y_{(L)}\right) = 0$.	**0.25** **0.25**
3)		• $D_f = \mathbb{R}$ centered at 0 and $f(-x) + f(x) = -1$.	**0.75**
4)	**a-**	• $f'(x) = \dfrac{e^{2x} + e^x + 1}{\left(e^x + 1\right)^2}$ • $f'(x) > 0$ since $e^x > 0$	**0.5** **0.25**
	b-	**x**: $-\infty$ … $+\infty$ **f '(x)**: + **f(x)**: $-\infty$ ↗ $+\infty$	**0.5**
5)	**a-**	$y = \dfrac{3}{4}x - \dfrac{1}{2}$	**0.5**
	b-	• $f(0) = -\dfrac{1}{2}$; then E belongs to (C). (T) tangent to (C) at E, then E belongs to (T) • $f(x) - y_{(T)} = \dfrac{1}{4}x - \dfrac{e^x}{e^x + 1} + \dfrac{1}{2}$; (C) cuts (T) at E; when $x < 0$: (C) below (T); when $x > 0$: (C) above (T); when $x = 0$: (C) cuts (T)	**0.5** **0.5**
6)		$f''(x) = \dfrac{e^x(e^x - 1)}{(e^x + 1)^3}$; at 0: $f''(0) = 0$ and $f''(x)$ changes signs (– to +).	**0.75**
7)		On $\mathbb{R}$: f is continuous, strictly increasing, and changes signs from $-\infty$ to $+\infty$, then $f(x) = 0$ has a unique root α. $f(0.6) = -0.04 < 0$ and $f(0.7) = 0.03 > 0$; $f(0.6) \times f(0.7) < 0$; $0.6 < \alpha < 0.7$.	**0.75**

8)			**1** **(C)**
	a-	Done before	**0.5**
	b-	$\begin{cases} y = \frac{3}{4}x - \frac{1}{2} \\ y = x \end{cases}$; y = y; x = –2 and y = –2.; F(–2 , –2)	**0.5**