I- (4 points)

The RENT A VEHICLE shop has two types of American and German vehicles in its store: cars and jeeps. 60% of the vehicles are cars out of which 70% are German and 80% of the jeeps are German.

Part A

One vehicle is chosen randomly from this shop.

1) Calculate the probability that the chosen vehicle is an American car.
2) Calculate the probability that the chosen vehicle is a car, knowing that it is American.

Part B

The owner of the shop states that he has 50 vehicles in the store. Maher, a customer, comes to rent two vehicles randomly and simultaneously from this store.

Consider the following events:

C_1: "The chosen vehicles are one car and one jeep."

C_2: "The chosen vehicles are cars."

A: "The chosen vehicles are American"

1) Calculate the following probabilities: $P(A / C_2)$ and $P(C_2 \cap A)$.
2) Let X be the random variable that designates the number of cars rent by Maher.
   - **a-** Prove that $P(X = 1) = \frac{24}{49}$.
   - **b-** Find the probability distribution of X.

II- (4 points)

The space is referred to a direct orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$. Consider the points A (1 ; 1 ; 1), B(2 ; 0 ; – 2), and C(3 ; 2 ; 1). Let (D) be the straight-line of a system of parametric equations: $\begin{cases} x = t + 1 \\ y = 2t + 3 \\ z = 3t + 2 \end{cases}$ $(t \in IR)$.

1) Calculate the area of the triangle ABC.
2) Verify that an equation of the plane (Q) determined by the three points A, B, and C is $x - 2y + z = 0$.
3) Write an equation of the plane (R) determined by the point A and the straight-line (D).
4) Find a system of parametric equations of the straight-line (L), the intersection of the two planes (Q) and (R). Verify that (Q) and (R) are perpendicular.
5) Calculate the distances from point E(1 ; 0 ; 4) to plane (Q) and to plane (R), then deduce the distance from E to line (L).

III- (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A, B, E, M, and M′ of respective affixes i, –i, 2 + i, z, and z′ such that $z' = \frac{z-i}{z+i}$ $(z \neq -i)$.

1) Let z = x + iy and z′ = x′ + iy′.
   a- Express x′ and y′ in terms of x and y.
   b- Find the set of points M when M′ moves on the straight-line of equation y′ = x′.
2) In this part, suppose that M′ moves on the circle of center O and radius 1.
   a- Find the set (L) of points M.
   b- Verify that F, the midpoint of [BE], belongs to (L).
3) Calculate $\frac{z_E - z_A}{z_B - z_A}$, then deduce the nature of triangle ABE.

IV- (8 points)

Part A

Consider the differential equation (E): xy′ – y = x – 1. Let y = xz.

1) Form the differential equation (F) satisfied by z, then solve it.
2) Deduce the general solution of (E), and the particular solution whose representative curve (C), in an orthonormal system $(O;\vec{i},\vec{j})$, passes through the point A(1 , 0).

Part B

Let f be a function defined, on $]0, +\infty[$, by: f(x) = xlnx – x + 1. Let (C) be the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$. (Graph unit: 2 cm).

1)
   a- Study the variations of f and construct its table of variations.
   b- Deduce the sign of f(x).
2) Prove that the equation f(x) = 2 admits a unique root α. Verify that 3.5 < α < 3.6 .
3) Let (d) be the straight-line of equation y = –x + 1. Study the relative positions of (d) and (C).
4) Draw (C).
5) Let (D) be the domain bounded by the curve (C), the line (d), and the two lines of equations x = 1 and x = α.
   a- Calculate, in terms of α, the area A(α) of (D).
   b- Prove that $A(\alpha) = (\alpha + 1)^2\ cm^2$.
6) Let F be the function defined, on $]1, +\infty[$, by: $F(x) = \int_1^x f(t)dt$. Without finding the explicit form of F(x), prove that F admits an inverse function.

Answer Key

Question I		Mark
Part A		
1)	$P(A \cap C) = \frac{30}{100} \times \frac{60}{100} = \frac{9}{50}$	**0.5**
2)	$P(C / A) = \frac{P(A \cap C)}{P(A)} = \frac{0.18}{P(A \cap C) + P(A \cap \overline{C})} = \frac{0.18}{0.18 + 0.8} = \frac{9}{13}$	**0.5**
Part B		
1)	$P(A / C_2) = \frac{C_9^2}{C_{30}^2} = \frac{12}{145}$ $P(C_2 \cap A) = \frac{36}{1225}$	**0.5** **0.5**
2) a-	$P(X = 1) = \frac{C_{30}^1 \times C_{20}^1}{C_{50}^2} = \frac{24}{49}$	**0.5**
2) b-	$X_\Omega = \{0 , 1 , 2\}$ $P(X = 0) = \frac{C_{20}^2}{C_{50}^2} = \frac{38}{245}$; $P(X = 2) = \frac{C_{30}^2}{C_{50}^2} = \frac{87}{245}$	**0.5** **0.5** **0.5**

Table for Part B 1):

	A	G	
C	9	21	30
J	4	16	20
	13	37	50

	Question II	Mark
1)	$\overrightarrow{AB} \wedge \overrightarrow{AC} = 3\vec{i} - 6\vec{j} + 3\vec{k}$; Area = $\frac{3\sqrt{6}}{2}$ units of area	**0.75**
2)	(Q): $\overrightarrow{AM} \cdot (\overrightarrow{AB} \wedge \overrightarrow{AC}) = 0$; $x - 2y + z = 0$	**0.5**
3)	(R): $4x + y - 2z - 3 = 0$	**0.75**
4)	(L): $\begin{cases} x = \frac{1}{3}m + \frac{2}{3} \\ y = \frac{2}{3}m + \frac{1}{3} \\ z = m \end{cases}$ ($m \in R$). $\vec{N}_Q \cdot \vec{N}_R = 4 - 2 - 2 = 0$; (Q) and (R) are perpendicular	**0.75** **0.25**
5)	$d(E , (Q)) = \frac{5\sqrt{6}}{6}$ u and $d(E , (R)) = \frac{\sqrt{21}}{3}$ u $d(E , (L)) = \sqrt{(d(E,(Q)))^2 + (d(E,(R)))^2} = \frac{\sqrt{26}}{2}$ u	**0.25** **0.25** **0.5**

		Question III	Mark
1)	a-	$x' = \dfrac{x^2+y^2-1}{x^2+(y+1)^2}$ and $y' = \dfrac{-2x}{x^2+(y+1)^2}$	1
	b-	$y' = x'$; then $x^2 + y^2 - 1 = -2x$ and $x^2 + (y+1)^2 \neq 0$; then $(x+1)^2 + y^2 = 2$; M moves on the circle of center $(-1 , 0)$ and radius $\sqrt{2}$ deprived of point B	1
2)	a-	$OM' = 1$; $MA = MB'$; M moves on the perpendicular bisector of [AB]	0.75
	b-	$F(1 , 0)$; $FA = FB = \sqrt{2}$; F belongs to (L)	0.5
3)		$\dfrac{z_E - z_A}{z_B - z_A} = i$; $\dfrac{AE}{AB} = 1$ and $\left(\overrightarrow{AB};\overrightarrow{AE}\right) = \dfrac{\pi}{2}$; ABE is right-isosceles	0.75

		Question IV	Mark
Part A			
1)		(F): $z' = \dfrac{1}{x} - \dfrac{1}{x^2}$. $z = \ln\lvert x\rvert + \dfrac{1}{x} + C$	0.5 0.5
2)		G.S.: $y = x\ln\lvert x\rvert + 1 + Cx$; $y = x\ln\lvert x\rvert - x + 1$	0.25 0.5
Part B			
1)	a-	$\begin{array}{c\|ccccc} x & 0 & & 1 & & +\infty \\ \hline f'(x) & & - & 0 & + & \\ \hline f(x) & 1 & \searrow & 0 & \nearrow & +\infty \end{array}$	1
	b-	$f(x) \geq 0$ since its minimum is zero	0.25
2)		On $]1 , +\infty[$: f is continuous, f is strictly increasing, $2 \in]0 , +\infty[$; then the equation $f(x) = 2$ admits a unique root α. But $f(3.5) = 1.8 < 2$ and $f(3.6) = 2.01 > 2$, then $3.5 < \alpha < 3.6$	1
3)		$f(x) - y_d = x\ln x$: if $x \in]0 , 1[$, then (C) is below (d); if $x = 1$, then (C) cuts (d); if $x \in]1 , +\infty[$, then (C) is above (d)	0.5
4)		Curve	1
5)	a-	$A(\alpha) = \left(\dfrac{\alpha^2}{2}\ln\alpha - \dfrac{\alpha^2}{4} + \dfrac{1}{4}\right)$ units of area.	1
	b-	$f(\alpha) = 2$; $A(\alpha) = \left(\dfrac{\alpha}{2} + \dfrac{\alpha^2}{4} + \dfrac{1}{4}\right) \times 4cm^2 = (\alpha + 1)^2 cm^2$	1
6)		On $]1 , +\infty[$: $F'(x) = f(x) > 0$. On $]1 , +\infty[$: F is continuous and strictly increasing, then it admits an inverse function	0.5