Class: Third secondary: Life Sciences

Subject: Mathematics

First Unified Exam

Exercise 1(4pts)

Choose and justify the correct answer :

1) If $z=\sqrt{3}+i$, then z^{12} is :
   a. 2^{12} b. $2^{12}\,i$ c. -2^{12}.

2) If $(z-1)(\bar{z}-i)$ is real , then the set of points M(z) is :
   a. a circle b. a straight line c. a semi straight line.

3) In the equation $z + i\bar{z} = (1+i)$, the number z is equal to:
   a. $2+i$ b. $2-i$ c. $-2+i$.

4) If $z=\dfrac{3+4i}{1-2i\sqrt{6}}$, then $\left|z^{10}\right|+\left|\bar{z}^{10}\right|$ is equal to :
   a. 1 b. 2 c. 2^{10}

Exercise 2:(2pts)

A and B are two events of the same sample space such that $p(A)=0.1$, $p(B)=0.3$ and $p(A\cup\bar{B})=0.73$.

1. Prove that $p(A\cap\bar{B})=0.07$ and show that $p(A/B)=p(A)$
2. Calculate $p(\bar{A}\cap B)$ and $p(\bar{A}\cup B)$

Exercise 3:(7pts)

An urn contains 5 black balls and 3 white balls. Two balls are drawn randomly and simultaneously from this urn.

For each black ball drawn, Ali wins \$ 300.

For each white ball drawn, Ali loses \$ 400.

let X denotes the random variable corresponding to the sum of money (positive or negative).

1) Determine the probability distribution of X.
2) Calculate the expectation E (X). Interpret the obtained result.
3) We throw a coin.

If you get the tail, add a white ball to the box.

If we get the head, a black ball is added to the box. Three balls are drawn randomly and simultaneously .

consider the two events :

A <<a single ball drawn is white>>

B <<This gives the tail>>

a) Show that $P[A/B]=\dfrac{10}{21}$ and deduce $P[A\cap B]$

b) Calculate $P[A\cap \overline{B}]$ and deduce $P[A]$

<u>Exercise 4:(7pts)</u>

The complex plane is referred to a direct orthonormal system (R) = (O, $\vec{u}$, $\vec{v}$) of graphic unit 2 cm. Consider the points A and B of respective affixes $z_A = 1 + i\sqrt{3}$ and $z_B = 1 - i\sqrt{3}$.

1-Write the exponential form of z_A and deduce that of z_B. Construct the points A and B in the system (R).

2- M(z) and M'(z') are points of the complex plane so that $z' = e^{i\frac{2\pi}{3}} z$.

a. Verify that $z' = \dfrac{1}{2}(-1 + i\sqrt{3})\, z$. Write the algebraic and exponential forms of z' when

$z = -1 - i$.

Deduce the exact value of $\cos\dfrac{\pi}{12}$.

b. Find z' in the case where $z = z_A$. The value of z' thus found will be the affix of a point C.

c. What is the nature of triangle ABC? Justify your answer.

Exercise I:

1) $(\sqrt{3}+i)^{12} = (2e^{i\frac{\pi}{6}})^{12} = 2^{12}e^{i2\pi} = 2^{12}$

2) Consider the points A and B of affixes 1 and $-i$ respectively.

We have $\arg(z-1)+\arg(\bar{z}-i) = k\pi$

$\arg(z-1)-\arg(z+i) = k\pi$ that is $(\vec{BM}, \vec{AM}) = k\pi$

This means that the set of points of M is a straight line.

3) Take $z = x+iy$ so, we have: $x+iy+i(x-iy) = 1+i$

$x+iy+ix+y = 1+i$ that is $(x+y)+i(x+y) = 1+i$

We have one equation $x+y=1$ and only the point $(2,-1)$ satisfy this equation

The correct answer is $2-i$

4) $|z| = \left|\dfrac{3+4i}{1-2i\sqrt{6}}\right| = \dfrac{|3+4i|}{|1-2i\sqrt{6}|} = \dfrac{5}{5} = 1$ and $|\bar{z}| = |z| = 1$

Then $|z^{10}| + |\bar{z}^{10}| = |z|^{10} + |\bar{z}|^{10} = 1+1 = 2$

Exercise II:

1. We have $p(A\cup\bar{B}) = p(A)+p(\bar{B})-p(A\cap\bar{B})$ with $p(\bar{B}) = 1-p(B) = 0.7$

We get $p(A\cap\bar{B}) = p(A)+p(\bar{B})-p(A\cup\bar{B}) = 0.1+0.7-0.73 = 0.07$

Also, we have $p(A) = p(A\cap B)+p(A\cap\bar{B})$ to get that $p(A\cap B) = 0.1-0.07 = 0.03$

$p(A/B) = \dfrac{p(A\cap B)}{p(B)} = \dfrac{0.03}{0.3} = 0.1 = p(A)$

2. We have $p(B) = p(A\cap B)+p(\bar{A}\cap B)$ to get that $p(\bar{A}\cap B) = 0.3-0.03 = 0.27$

Also, $p(\bar{A}\cup B) = p(\bar{A})+p(B)-p(\bar{A}\cap B) = 0.9+0.3-0.27 = 0.93$

Exercise III:

1) $X = \{-800, -100, 600\}$

$p(x=-800) = p(2white) = \dfrac{C_3^2}{C_8^2} = \dfrac{3}{28}$ $\qquad$ $p(x=600) = p(2black) = \dfrac{C_5^2}{C_8^2} = \dfrac{10}{28} = \dfrac{5}{14}$

$p(x=-100) = p(1white1black) = \dfrac{C_3^1C_5^1}{C_8^2} = \dfrac{15}{28}$

2) $E(X) = 600\times\dfrac{10}{84} - 800\times\dfrac{3}{28} - 100\times\dfrac{15}{28} = \dfrac{2100}{28} = 75$

The average amount of money Ali earned is 75$

3) a) $p(A/B) = \dfrac{C_4^1\times C_5^2}{C_9^3} = \dfrac{40}{84} = \dfrac{10}{21}$

$$p(A\cap B)=p(A/B)\times p(B)=\frac{10}{21}\times\frac{1}{2}=\frac{5}{21}$$

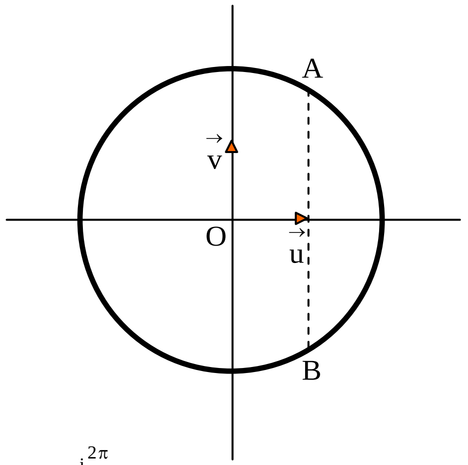
b) $p(A\cap \overline{B})=p(A/\overline{B})\times p(\overline{B})=\dfrac{C_3^1\times C_6^2}{C_9^3}\times\dfrac{1}{2}=\dfrac{45}{168}$

so, $p(A)=p(A\cap B)+p(A\cap \overline{B})=\dfrac{5}{21}+\dfrac{45}{168}=\dfrac{85}{168}$

Exercise IV:

1. $Z_A = 1 + i\sqrt{3} = 2(\frac{1}{2}+i\frac{\sqrt{3}}{2}) = 2e^{i\frac{\pi}{3}}$

Since Z_B is the conjugate of z_A , so $Z_B = 2e^{-i\frac{\pi}{3}}$.
A and B belong to the same circle of center O and radius 2, and they are on the line of equation x = 1.



2. $z' = e^{i\frac{2\pi}{3}} z$

a. $z' = [\cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3})]z = (-\frac{1}{2}+i\frac{\sqrt{3}}{2})z$

if $z = -1 - i$, then $z' = (-\frac{1}{2}+i\frac{\sqrt{3}}{2})(-1-i)$

$$= \frac{1+\sqrt{3}}{2} + \frac{1-\sqrt{3}}{2}i$$

$Z' = e^{i\frac{2\pi}{3}} \times \sqrt{2}e^{i\frac{5\pi}{4}} = \sqrt{2}e^{i\frac{23\pi}{12}} = \sqrt{2}e^{-i\frac{\pi}{12}}$

$= \sqrt{2}[\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}]$

Value of $\cos(\frac{\pi}{12}) = ?$

By comparison, we get $\sqrt{2}\cos\frac{\pi}{12} = \frac{1+\sqrt{3}}{2}$

and $\cos\frac{\pi}{12} = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}$.

b. $z_C = \frac{1}{2}(-1+i\sqrt{3})(1+i\sqrt{3}) = -2$

c. $\dfrac{z_C - z_A}{z_C - z_B} = \dfrac{-3-i\sqrt{3}}{-3+i\sqrt{3}} = \dfrac{\sqrt{3}+i}{\sqrt{3}-i} = \dfrac{(\sqrt{3}+i)^2}{4}$

$= \dfrac{3-1+i\sqrt{3}}{4} = \dfrac{1}{2}+i\dfrac{\sqrt{3}}{2} = e^{i\frac{\pi}{3}}$

so we get $\frac{CA}{CB}\ e^{i(\overrightarrow{CB},\overrightarrow{CA})} = e^{i\frac{\pi}{3}}$

ABC : equilateral triangle

Remark : (Ask your teacher about another way)