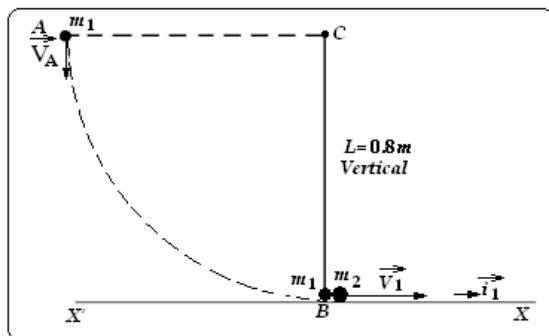


MID YEAR EXAM

Answer the following questions:

I- Conservation and non conservation of mechanical energy- collision(7 pts.)

Consider two point masses $m_1 = 0.1\text{Kg}$ and $m_2 = 0.4\text{Kg}$. The point mass m_1 is fixed to one end "A" of a mass less, inextensible string CA, whose other end is tied to a fixed support C. The length of the string is $L = 0.8\text{m}$. The point mass m_2 rests on a horizontal surface $x'x$, taken as zero reference G.P.E.



m_1 taken to position "A" of same level as C, and with $CA=L$, m_1 is given a vertical downward velocity \vec{V}_A , thus m_1 reaches B with a velocity \vec{V}_1 , of magnitude $V_1 = 5\text{m/s}$. ($\vec{V}_1 = 5\vec{i}$) m_1 enters into head on collision with m_2 , which is originally at rest.

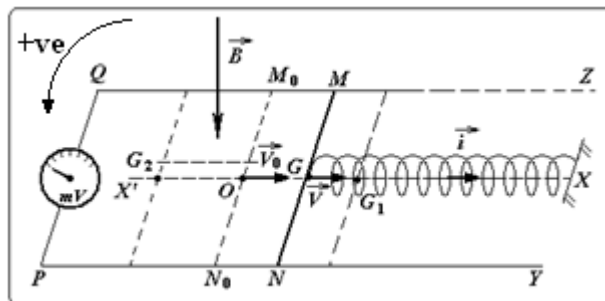
Just after collision m_2 moves with velocity \vec{V}_2' , of magnitude 2m/s , ($\vec{V}_2' = 2\vec{i}$) and m_1 moves with velocity \vec{V}_1' . Neglect air resistance. Use $g = 10\text{m/s}^2$.

- 1- Explain why the mechanical energy of the system (m_1 , earth) along the path \widehat{AB} is conserved, then determine its value, and deduce the magnitude of \vec{V}_A .
- 2- Determine \vec{V}_1' . Is the collision between m_1 and m_2 perfectly elastic? Why?
- 3- In what direction and with what maximum angle would the string CB deviate after collision.
- 4- After collision, m_2 starts with the speed 2m/s , and stops on the horizontal surface $x'x$ along a distance of 5m , due to friction force \vec{f} supposed constant.
 - a) Find f .
 - b) Determine G.P.E, M.E, and K.E of m_2 in terms of x . Where x represents the position of m_2 with origin B. Draw the graphs representing G.P.E, M.E, and K.E versus x .

II- Horizontal Elastic pendulum - Electromagnetic induction(7 pts.)

In the figure: the horizontal spring is mass less, of stiffness $K = 50\text{N/m}$. The rod MN is uniform, homogeneous, of mass 0.5Kg , and length $L = 0.25\text{m}$.

One end of the spring is fixed; the other end holds MN through its center G. The rod MN is a conductor free to slide on the horizontal wires QZ and PY, with MN perpendicular to them. The millivoltmeter has very high resistance. The whole circuit MNPQ lies in a horizontal plane. The rod MN, the rails QZ, PY, and all connecting wires are of negligible resistance. QZ and PY are parallel, to the axis $x'x$ of the spring, QZ is the internal wire.



At $t_0 = 0$, the spring has its free length, the rod is at position M_0N_0 is given a velocity $\vec{V}_0(m/s) = 2\vec{i}$. \vec{i} is a unit vector along $x'ox$. The zero reference G.P.E is level of $x'x$. At any t , $\vec{OG} = x$ and $\vec{V} = x'\vec{i}$. friction is negligible.

- 1- Write the expression of the M.E of the system (rod MN, spring).
- 2- Using the principle of conservation of M.E, determine the differential equation of motion for G, then deduce the proper angular frequency ω_0 and proper period T_0 of this elastic pendulum.
- 3- Determine the time equation of motion of G. $x = ?$ in terms of t .
- 4- The whole circuit is subjected to a uniform magnetic field \vec{B} , vertically downward, of magnitude 0.4T.

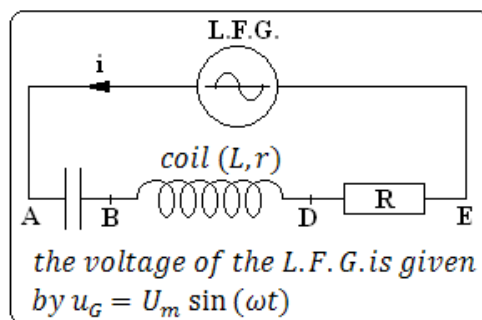
Denote by S_0 the surface area M_0N_0PQ , and by S the surface area of $MNPQ$.

- a) Determine the area of MNN_0M_0 or ΔS in terms of x , then deduce the magnetic flux Φ in terms of S_0 and x .
- b) Determine the induced e.m.f across MN in terms of V , and calculate its value e_0 at $t_0 = 0$.
- c) Do we have induced current I_0 ? Why?
- d) In case the millivoltmeter is replaced by a resistor $R=5\Omega$. Find the induced current I_0 at $t_0 = 0$, and the electromagnetic force \vec{F}_0 in M_0N_0 at same instant. Is the oscillation of MN in this case damped or undamped? Why? What must we do for MN to produce S.H.M, and what do we call such motion?

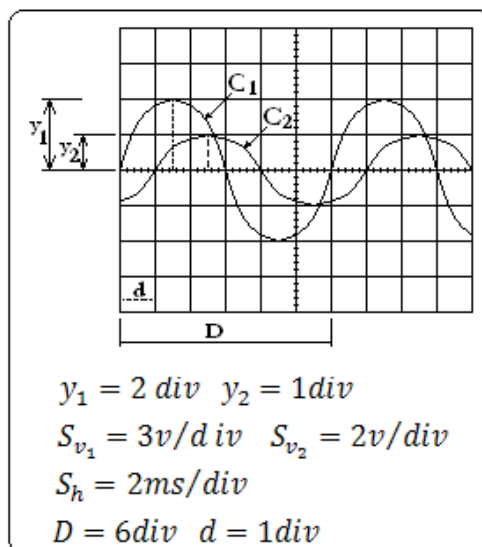
III- Study of sinusoidal alternating current – voltage.....(6pts.)

The adjacent circuit consists of a low frequency generator connected in series to a capacitor, of capacitance $C= 50\mu F$, a coil (L,r) and a resistor of resistance $R= 20\Omega$.

An oscilloscope is connected to the circuit, to display the voltage u_G across L.F.G; u_R across R so we get the curves (C_1) and (C_2) in the oscillograms.

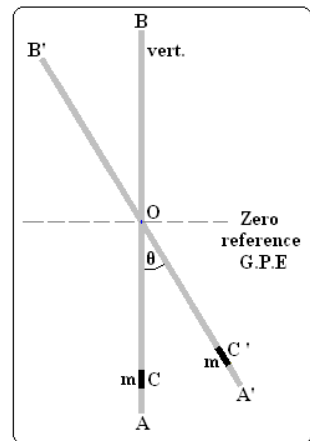


- 1- a) Calculate the amplitude $U_{m(1)}$ and $U_{m(2)}$ of the displayed voltages, then deduce which curve represents u_G and draw the connections of the circuit to the oscilloscope.
- b) Calculate the amplitude of the current I_m , the period of u_G , and the phase angle φ between u_G and i , indicating which one leads the other, then deduce u_G and i in terms of t
- c) Find the average power across AE of the circuit, then deduce r .
- 2- The frequency of the L.F.G varies, and when $u_G = 6 \sin(200\pi t)$ or $f_0 = 100 \text{ Hz}$, we observe that u_G and u_R are in phase. (u_G in V, t in S)
 - a) What do we call this phenomenon? Calculate the inductance L of the coil.
 - b) Find the new amplitude I'_m of the current, then deduce the expression of the current i' in terms of t .
 - c) Determine the expressions of the voltages $u_{(coil)}$ and u_C in terms of time.



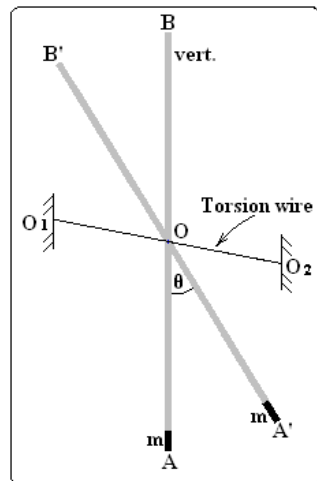
IV- Study of Compound and Torsion Pendulums.....(7.5pts.)

- 1- Consider a uniform, homogenous rod AB, of length $L=1\text{m}$, and mass $M=0.6\text{Kg}$, free to oscillate without friction about a horizontal axis (Δ), passing through the mid-point O of AB. A point mass $m=0.2\text{Kg}$ is connected to AB at C, with $\overline{OC} = x$, C is between O and A. The horizontal plane passing through O is taken a zero reference G.P.E. The moment of inertia of this rod about the axis (Δ) is given by $I_0 = \frac{1}{12}ML^2$. The pendulum (m, rod) is displaced round O by angle $\theta_m = 0.1\text{rd}$ and released from rest at $t_0 = 0$. Denote by G the center of mass of this compound pendulum. $\overline{OG} = a$.



Use: $g = 10\text{m/s}^2$, $\sin \theta = \theta$, $\cos \theta = 1 - \frac{1}{2}\theta^2$, θ in rd.

- Determine "a" in terms of x .
 - Determine the moment of inertia I of the system (m, rod AB) relative to the axis (Δ), in terms of x .
 - During the oscillations of this pendulum, when passing by position θ , with angular speed θ' .
 - Determine the M.E of this pendulum in terms of θ , θ' , and x .
 - Determine the differential equation describing this motion, then deduce the proper period T_0 of this pendulum in terms of x .
 - What value should be given to x in order to get a minimum value for T_0 , and show that $T_{0(\min)} = 2\text{S}$.
 - Determine x when $T_0 = 2.5\text{S}$.
- 2- Now m is fixed at A, and the system (m, rod) is free to oscillate about a horizontal torsion wire O_1O_2 passing through O. At zero torsion AB is vertical- The system (m, rod AB) is given a deviation $\theta_m = 0.1\text{rd}$ about O_1O_2 and released from rest. The horizontal plane containing O is taken zero reference G.P.E, when AO makes angle θ with the vertical, it has angular speed θ' .



- Determine the total potential energy of the system (m, rod AB, torsion wire O_1O_2) in terms of θ and the constant of torsion C.
- Determine the M.E of the system (m, rod AB, torsion wire O_1O_2) in terms of C and θ , then find the differential equation of motion of this pendulum (Neglect friction).
- Using part (b), find the proper period of this pendulum in terms of C.
- Knowing that the period T_0 of this pendulum is 1.8S , calculate C.

Good work

