

FINAL EXAM

Non programmable Calculators are allowed

First exercise: (6points)

Determination of an unknown component

In order to identify an electric component D and find its characteristic quantity $[x]$, we connect it in series with a resistor $R=100\Omega$ across a LFG delivering the voltage $u_G = 12\cos(100\pi t)$ (u in volt, t in sec).

This unknown component D may be a resistor (x is then its resistance) or a capacitor (x is then its capacitance) or a coil of negligible resistance (x is then its inductance). When the channels Y_1 and Y_2 of the oscilloscope are connected as shown, we obtain the waveforms shown in figure 1.

The vertical sensitivity on the channel Y_2 is 3v/div .

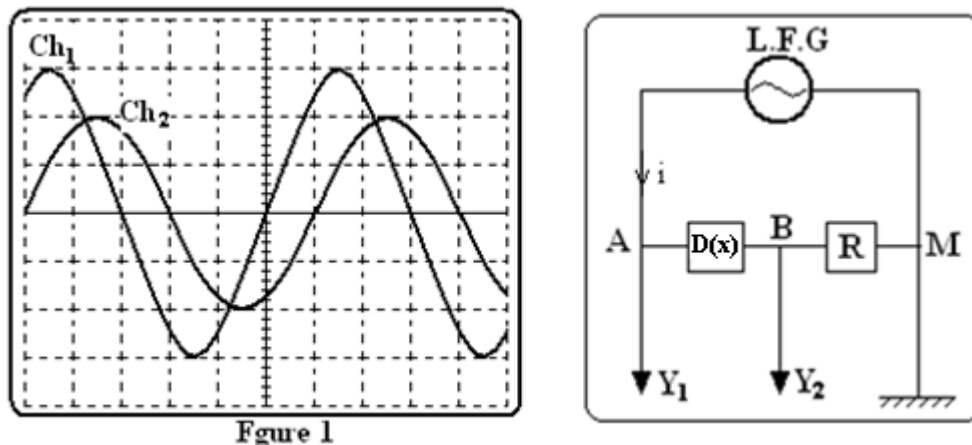


Figure 1

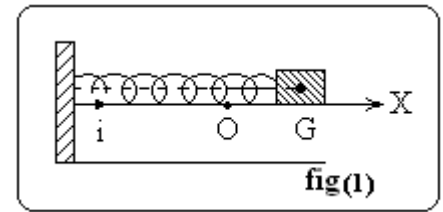
- 1) Calculate the vertical sensitivity on the channel Y_1 .
- 2) Which waveform leads the other? And by what angle? Deduce the nature of the unknown component.
- 3) Write the expression of the current i in the circuit as a function of time and deduce the average power consumed in the circuit.
- 4) Determine $u_D = u_{AB}$ in terms of x and t . Then using the principle of addition of voltages, write the relation among u_G , u_D , and u_R . Using a particular value of t calculate x .
- 5) It is required to add a component in series with the above circuit that would make the effective current in the circuit attain a maximum value (Resonance current). Which of the following choices will do: (giving all the necessary explanations)
 - A resistor of resistance R'
 - A coil of inductance L' and of negligible resistance.
 - A capacitor of capacitance C' .

Calculate then the maximum effective current and the characteristic value of the chosen component.

Second exercise: (7points)

Mechanical Oscillator

An elastic horizontal pendulum is formed of a solid of mass $m=250\text{g}$ and a spring of stiffness $k=10\text{N/m}$. To study the motion of the center of mass G of the solid, an axis is chosen whose origin O coincides with the equilibrium position G_0 of the solid. $\overline{OG} = x$



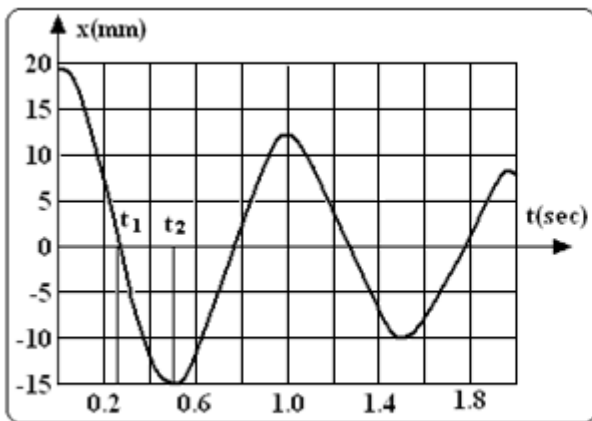
A) Friction is negligible.

Displace the solid 2cm from its equilibrium position and release it without initial velocity at $t=0$.

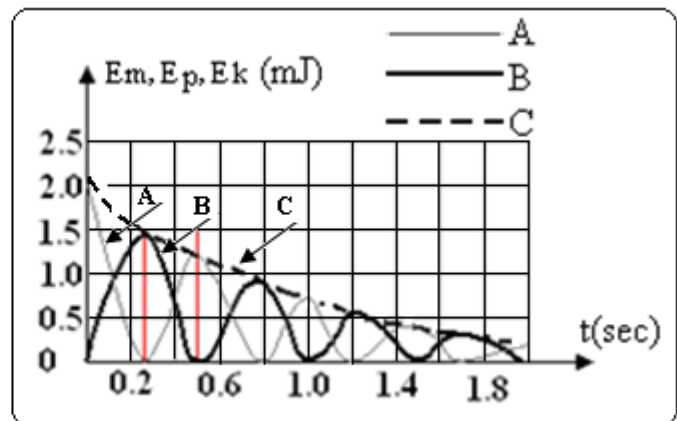
- 1- What is the mode of the oscillations formed?
- 2- Establish the differential equation that describes the motion of G .
- 3- Knowing that $X=X_m \cos(\omega_0 t + \varphi)$ is a solution of the differential equation, determine ω_0 in terms of k and m .
- 4- Determine the numerical values of X_m , ω_0 and φ . Deduce the proper period T_0 of the motion.

B) Friction is not negligible.

In reality the force of friction isn't negligible and equals to $\vec{f} = -k_1 \vec{V}$ where $k_1 = \text{cst} > 0$. The tracing of the position of G and the energies of the system (pendulum, Earth) as a function of time gives the following figures:



figure(2)



figure(3)

- 1) What is the mode of oscillations of the pendulum?
 - 2) Using figure (2), determine the pseudo-period T of the oscillations, and compare it with T_0 . Justify.
 - 3) Using figure (3), identify graphs A, B and C. find the period T_1 of A & B. Compare it with T .
 - 4) Two instants t_1 and t_2 are shown on figure (2). Which of the two speeds corresponds to?
 - a. Maximum speed,
 - b. zero speed? Justify.
 - 5) Determine the variation of the ME of the pendulum between the two instants $t_0 = 0$ and $t = 1.4\text{s}$.
 - 6) Calculate the work done by friction between these two instants.
- C) A driving force \vec{F} is applied on G so that the motion of G is described by $X=X_m \cos(\omega_0 t + \varphi)$ as in part A.
- 1) What is the mode of oscillations of the pendulum?

- 2) Determine \vec{F}_1 as a function of k_1 and t .
- 3) Determine the average power given to the pendulum between $t_0 = 0$ and $t = 1.4s$.

Third exercise: (7points)

The nuclide ${}^{108}_{47}Ag$ is β^- emitter.

1. a) Write the equation for this nuclear reaction, specifying the rules used.
- b) Specify the symbol of the resultant nucleus and give its nuclear composition. Using the following table:

${}_{46}Pd$	${}_{47}Ag$	${}_{48}Cd$
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- c) Calculate the energy liberated by the above reaction, knowing that:

$$m_{{}^{108}_{47}Ag} = 107.9865u, \quad m_{{}^{108}_{48}Cd} = 107.9615u, \quad m_{{}^{108}_{46}Pd} = 107.9050u, \quad m_{-1e} = 5.5 \times 10^{-4}u$$

$$1u = 931.5MeV/c^2.$$

2. a) Write the formula representing the law of radioactive decay (N in terms of t) and describe the meaning of each term. No need to demonstrate the formula.
- b) Define the radioactive period T.
- c) Establish the expression of the radioactive constant λ as a function of T.
- d) The variation in activity of a sample of ${}^{108}_{47}Ag$ is studied over time.

Activity A is defined by $A = -\frac{dN}{dt}$ and expressed in Becquerel's.

(One Becquerel corresponds to one disintegration per second).

- i) - Derive activity A as a function of time.
- Complete the following table:

T(min).....	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
A(Bq).....	90	73	63	52	44	39	33	29	24	21	18
lnA.....											

- ii) Draw the representative curve $\ln A = f(t)$,

Scales: Abscissa: $1cm \cong 0.5min$;

Ordinate: $1cm \cong 0.5$.

- iii) Using the above graph, Determine the radioactive constant λ of ${}^{108}_{47}Ag$.

Deduce its radioactive period.

- e) What is the number of nuclei initially present in this sample?

Fourth exercise: (7.5points)

Transient state – Steady state

A- (R,L) series circuit

In the circuit of figure 1, $L=1H$, $R=1K\Omega$ and $E=10V$.

At the instant $t_0 = 0$, we close the switch K. At the instant t, the circuit carries a current i. An oscilloscope, conveniently connected, allows displaying the variations of the voltage $U_R = U_{BC}$ as a function of time (Figure 2).

- 1- Derive the differential equation describing the variations of the current i in terms of R,L,E and t.

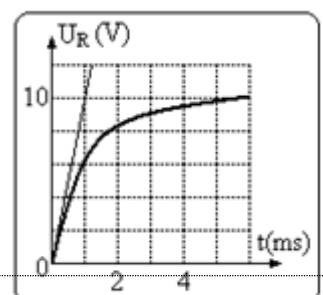
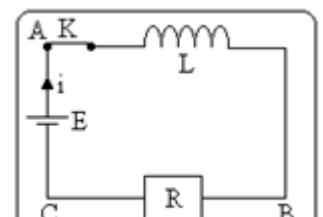


Figure 2

- 2- The solution of this equation is of the form: $i = A_1 - B_1 e^{\frac{-t}{\tau_1}}$. Determine the values of the constants A_1, B_1 and τ_1 and give the physical significance of each..
- 3- Referring to figure 2, verify that the values of τ_1 and A_1 are equal to those calculated above.
- 4- Determine:
 - a) The duration t_1 at the end of which the steady state is practically reached.
 - b) The value of the energy stored in the coil starting from t_1 .

B- A disk in a rotational motion

A disk can rotate a horizontal axis (Δ) that is perpendicular to its plane through its center O. The moment of inertia I of the disk with respect to (Δ) is $I = 1.52 \times 10^{-5} \text{kg.m}^2$.

When subjected to a motive couple, of constant moment $\mathcal{M}_M = 9.12 \times 10^{-3} \text{m.N}$, the disk starts to rotate from rest at the instant $t_0 = 0$. At the instant t, the physical quantities θ and θ' are respectively the angular abscissa and the angular velocity of the disk. During its rotation, the disk undergoes also a braking couple of moment $\mathcal{M}_F = -k\theta'$ where k is a positive constant of value $k = 3.04 \times 10^{-5}$ SI units.

Applying the theorem of angular momentum, show that the differential equation in θ' that describes the motion of the disk is written as: $I \frac{d\theta'}{dt} + k\theta' = \mathcal{M}_M$

C- An analogy

- 1- Match each of the physical electric quantities E, R, L, i, and $\frac{di}{dt}$ with the convenient mechanical physical quantity.
- 2-
 - a) Determine the solution of the differential equation in θ' .
 - b) Deduce the duration t_2 at the end of which the steady state is practically reached.
 - c) Determine the angular velocity in the steady state.

GOOD WORK