

I- (2 points)

In the table below, only one of the proposed answers to each question is correct. Write down the number of each question and give, with justification, the answer corresponding to it.

N°	Questions	Answers			
		a	b	c	d
1	The particular solution of the differential equation $y' - \frac{1}{2}y = 0$, that verifies $y(-2) = 1$, is:	$y = -2e^{\frac{x}{2}}$	$y = e^{\frac{x}{2} + 1}$	$y = 2\cos x - \sin x$	$y = \sqrt{x^2 - 3}$
2	$f(x) = 2\sin(\pi x + 2)$. The period of f is: $T =$	π	2	2π	$\frac{\pi}{2}$
3	The equation $2\ln x = \ln(2x)$ has :	2 roots	One root only	No roots	3 roots
4	If $f(x) = \ln -3x $; then $f'(x) =$	$\frac{3}{x}$	$-\frac{3}{x}$	$\frac{1}{ x }$	$\frac{1}{x}$
5	$e^{\frac{1}{2}\ln 9} \times e^{-\ln \frac{1}{3}} =$	e^3	6	$e^{\frac{3}{2}}$	9
6	$\cos^2\left(\frac{1}{2}\arccos x\right) =$	$\frac{1+x}{2}$	$1 + \frac{x}{2}$	$\frac{1}{2}x$	$(1+x)^2$

II- (3 points)

Given $(C_m): (m^2 - 1)x^2 + y^2 + 6my + 9 = 0$.

- 1) Discuss according to the values of m , the nature of (C_m) .
- 2) Determine the elements of (C_2) (corresponding to $m = 2$).
- 3) Find the equations of the tangent and normal to (C_2) at $L\left(\frac{\sqrt{6}}{3}; -1\right)$.
- 4) The tangent and normal cut $(x'Ox)$ at T and N.

Calculate $\overline{OT} \times \overline{ON}$. *Page 1 of 4*

III-(3 points)

In the complex plane referred to an orthonormal system $(O; \vec{u}; \vec{v})$, consider the points A, B, C and D of affixes $z_A = -2i$, $z_B = 4 - 2i$, $z_C = 4 + 2i$ and $z_D = 1$ respectively.

- 1) Locate the points A, B, C and D and specify the nature of triangle ABC.
- 2) Let f be the mapping which to every point M of affix z and distinct from A

assigns a point M' of affix $z' = \frac{z-4-2i}{z+2i}$.

a) Determine the images of B and C under f .

b) Determine the set (δ) of points M' of affix z' such that $|z'| = 1$.

- 3) Prove that for every point M distinct from A and whose image by f is M'

we have:

$$M' \neq D$$

$$\overline{DM'} \cdot \overline{AM} = 4\sqrt{2}$$

$$(\vec{u}; \overline{DM'}) + (\vec{u}; \overline{AM}) = \frac{5\pi}{4} \text{ rd.}$$

IV- (3 points)

To encourage national tourism, a tourist agency proposes week-ends of two days, and offers its customers three choices:

- Full-board week-end
- Half-board week-end
- Luxury week-end.

The agency published the following advertisement:

Choice \ Destination	Full-board	Half-board	Luxury
Mountain	150 000 LL	100 000 LL	200 000 LL
Beach	100 000 LL	75 000 LL	150 000 LL

This agency estimates that 65% of its customers choose mountains, and the others choose the beach; and that out of the customers to any destination 55% choose full-board and 30% choose half-board while the others choose luxury week-ends.

A customer is chosen at random and is interviewed.

Consider the following events:

A : « the interviewed customer has chosen the mountains ».

B : « the interviewed customer has chosen the beach ».

C : « the interviewed customer has chosen full-board week-end ».

D : « the interviewed customer has chosen half-board week-end ».

S : « the interviewed customer has chosen the luxury week-end ».

- 1) a- Calculate the following probabilities: $P(A \cap C)$, $P(B \cap C)$ and $P(C)$.
 b- The interviewed customer had chosen full-board, what is the probability that he chose the beach?
- 2) Let X be the random variable that is equal to the amount paid to the agency by a customer.
 - a- Show that $P(X=150\,000) = 0.41$ and determine the probability distribution for X .
 - b- Calculate the mean(expected value) $E(X)$. What does the number thus obtained represent?
 - c- Estimate the sum received by this agency when it serves 200 customers.

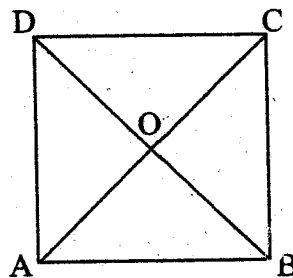
V- (3 points)

Consider, in an oriented plane, the direct square $ABCD$

with center O such that $(\vec{AB}, \vec{AD}) = \frac{\pi}{2} (2\pi)$.

Let r be the rotation with center O and angle $\frac{\pi}{2}$
 and h be the dilation (homothety) with center C
 and ratio 2.

Designate by S the transformation $r \circ h$.



- 1) Determine the nature of S and specify its ratio and its angle.
- 2) Designate by W the center of S .
 - a- Show that $S(C) = D$ and that $S(O) = B$.
 - b- Construct the point W , specifying clearly the steps of this construction.
- 3) The plane is referred to an orthonormal system $(A; \vec{AB}, \vec{AD})$.
 - a- Write the complex form of S and deduce the affix of the center W .
 - b- Determine the image of the square $ABCD$ under S .

VI- (6 points)

Part A

Let f be the function defined over \mathbb{R} by $f(x) = \frac{e^{2x} - 1}{e^{2x} + 1}$.

Designate by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

- 1) Study the parity of f .
- 2) Show that, for all $x \in \mathbb{R}$, $-1 < f(x) < 1$.
- 3) a) Calculate $\lim_{x \rightarrow -\infty} f(x)$ and $\lim_{x \rightarrow +\infty} f(x)$.
b) Deduce the equations of the asymptotes to (C).
c) Study the variations of f and draw its table of variations.
d) Show that f admits, over \mathbb{R} , an inverse function f^{-1} .
Determine the domain of definition of f^{-1} and calculate $f^{-1}(x)$.

Part B

- 1) Determine an equation of the tangent (T) to (C) at the point of (C) of abscissa 0.
- 2) a) Show that for all numbers t , $f'(t) = 1 - [f(t)]^2$.
b) Bound $f'(t)$.

c) For x being positive or zero, bound $\int_0^x f'(t) dt$.

Justify that $0 \leq f(x) \leq x$ and deduce the relative positions of (C) and (T).

- 3) B is a point of (C) of positive abscissa, determine B, knowing that the slope of the tangent to (C) at B is $\frac{1}{2}$.
- 4) Trace (C) and (T).
- 5) a) Show that $f(x)$ can be written in the form $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.
b) Calculate the area of the domain limited by (C), the straight line of equation $y = x$ and the two straight lines of equations $x = 0$ and $x = 1$.

Good Work