In this sample, the referred system is orthonormal unless stated.

I- (2 points)

In the table below only one, among the proposed answers to each question, is correct. Write down the number of each question and give, **with justification**, the corresponding answer.

	Questions	**Proposed Answers** A	B	C
1)	A solution of the differential equation (E): $y'' + 3y' - 4y = 0$ is	$e^{-x} + 2e^{4x}$	e^{4x}	e^{-4x}
2)	The negation of the statement $p \Rightarrow q$ is	$\neg p \Leftarrow \neg q$	$p \wedge (\neg q)$	$p \vee (\neg q)$
3)	Let $x > 0$. $\arctan x + \arctan \frac{1}{x} =$	$\frac{\pi}{2}$	0	$-\frac{\pi}{2}$
4)	$\int \frac{1}{x^2+4}dx =$	$\arctan(x)$	$\frac{1}{2}\arctan\left(\frac{x}{2}\right)$	$\frac{1}{4}\arctan\left(\frac{x}{2}\right)$
5)	Let f be a function defined by: $f(x) = e^{2x}$. $f^{(n)}(0) =$	2^{n-1}	2^{n}	2^{n+1}
6)	Let f be a continuous, differentiable, and strictly decreasing function on $\mathbb{R}$. The sequence $I_n = \int_n^{n+1} f(x)dx$, where $n \in N^*$, is	Increasing	Decreasing	None
7)	Let f be a continuous, differentiable, and positive function on $\mathbb{R}$. The sequence $I_n = \int_0^{n} f(x)dx$, where $n \in N^*$, is	Increasing	Decreasing	None
8)	Consider the following data: **Variable**: [10 – 15[, [15 – 20[, [20 – 25] **Frequency**: 7 , 3 , 10 The standard deviation of the above data is about	3.1	18.25	None

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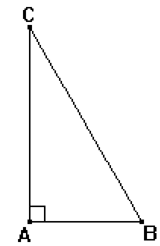
II- (3 points)

In an oriented plane, consider a direct triangle ABC right angled at A, such that AB = 2cm and $\left(\overrightarrow{BC}\ ;\ \overrightarrow{BA}\right)=\frac{\pi}{3}\ (2\pi)$. Let S be the direct similitude that transforms A onto B and B onto C.



1) Determine the ratio and the angle of S.

2) Let O be the midpoint [AB], and consider the direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$ such that $\vec{u}=\overrightarrow{OB}$.

a- Find the complex form of the similitude S.

b- Determine the affix of point W, the center of S.

3) Let E and F be two points on (AB) such that $\overrightarrow{EB}=2\overrightarrow{EA}$ and $\overrightarrow{FB}=-2\overrightarrow{FA}$.

a- Calculate $\overrightarrow{WE}$ and $\overrightarrow{WF}$ as a linear combination of (in terms of) $\overrightarrow{WA}$ and $\overrightarrow{WB}$.

b- Prove that W moves on a circle of diameter [EF].

c- Calculate WA, then deduce a position of W.

d- Specify, geometrically, the position of W.

4) Let H be the orthogonal projection of A on (BC). Consider the dilation h of center H that transforms B onto C.

a- Find the image of A by h.

b- Find the ratio and an angle of the similitude h o S.

III- (2 points)

Rami buys a 1000 LL bill that permits him to participate in a game. This game constitutes of rubbing the bill, followed by a lottery.

When Rami rubs the bill, he might gain 10,000 LL with a probability of 0.02 or gain nothing.

After, Rami participates in a lottery using the same bill. He might gain 10,000 LL, 20,000 LL or nothing regardless of whether he gained in rubbing the bill or not.

If Rami does not gain in rubbing the bill, the probability that he gains 10,000 LL in the lottery is $\frac{1}{70}$ and the probability that he gains 20,000 LL in the lottery is $\frac{1}{490}$.

Consider the following events:

G: "Rami gains in rubbing the bill."

L_0: "Rami gains nothing in the lottery."

L_1: "Rami gains 10,000 LL in the lottery."

L_2: "Rami gains 20,000 LL in the lottery."

1) Calculate the probability that Rami gains nothing in the lottery knowing that he did not gain anything in rubbing the bill.

2) Let X be the random variable that designates the total algebraic gain of Rami after he rubs the bill and the lottery. We know that P(X = 9,000 LL) = 0.016 and P(X = 19,000 LL) = 0.004 .

a- Prove that the probability that Rami gains 10,000 LL in the lottery given that he gained 10,000 LL in rubbing the bill is equal to 0.1 .

b- Determine the probability distribution of X and then calculate E(X).

c- Find the probability that Rami does not gain in the lottery knowing that he gained 10,000 LL in rubbing the bill.

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IV- (3 points)

The space is referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$. Consider the point A(–1 , 1 , 0), the plane (P) of equation: x – 2y + 2z – 6 = 0 and the straight-line (D): $\begin{cases} x = m+1 \\ y = 2m+1 \\ z = 3m+2 \end{cases}$ ($m \in \mathbb{R}$).

1) Prove that A and (D) determine a plane (Q) and determine an equation of (Q).

2)

a- Prove that (P) and (Q) intersect along the line (Δ) defined by: x = 2 ; y = t – 2 ; z = t.

b- Prove that the coordinates of A', the orthogonal projection of A on (Δ), are $\left(2\,,\,-\frac{1}{2}\,,\,\frac{3}{2}\right)$.

3) M is a variable point of (Δ). Let α be a measure of the angle that (AM) makes with (P).

a- Find the coordinates of H, the orthogonal projection of A on (P).

b- Calculate the distance from A to (P), then prove that AM × sinα = 3.

c- Determine the position of M so that α is maximum. Calculate sinα in this case.

d- What does this value of α represent for the two planes (P) and (Q)?

4) Consider the circle (C) of center A tangent to (Δ) and lying in the plane (Q). The orthogonal projection of (C) on the plane (P) is an ellipse (E).

a- Calculate the radius of (C).

b- Determine the coordinates of the center of (E).

c- Calculate the eccentricity of (E).

d- Determine a system of parametric equations of the focal axis of (E).

V- (3 points)

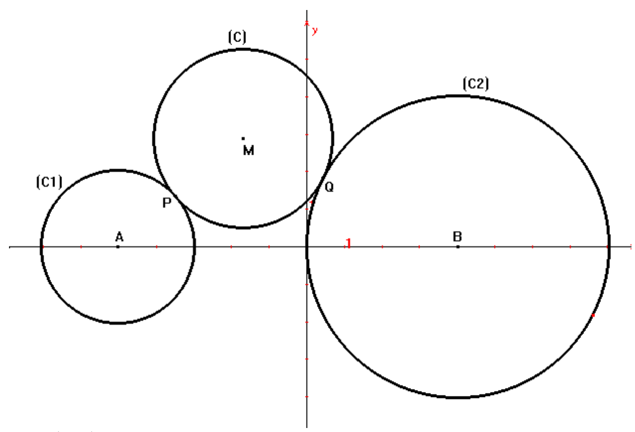
In the given figure, we have the circles:

(C_1) of center A(– 5 ; 0) and radius $R_1 = 2$;

(C_2) of center B(4 ; 0) and radius $R_2 = 4$;

(C) of center M and radius R and tangent externally to the circles (C_1) and (C_2) at points P and Q.

Consider the points C(–14 ; 0) and D(–2 ; 0).



1) Let S be the similitude of angle $\frac{\pi}{2}$ that transforms (C_1) to (C_2).

a- Calculate the ratio of S.

b- Let I be the center S. Use the equality $\left(\overrightarrow{IB}\right)^2 = 4\left(\overrightarrow{IA}\right)^2$ to prove that $\overrightarrow{IC}\cdot\overrightarrow{ID} = 0$, then construct geometrically the point I.

2) Show that M varies on a hyperbola (H) whose reduced equation and its eccentricity are to be determined.

3) Consider the two dilations h and h' such that $h(C_1) = (C)$ and $h'(C) = (C_2)$. Prove that $h' \circ h$ is a dilation and specify its elements.

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VI- (7 points)

Part A

Let f be a function defined, on $\mathbb{R}$, by: $f(x) = e^{-x}\ln(1 + e^{x})$. Let (C) be the representative curve of f in an orthogonal system $(O ; \vec{i} , \vec{j})$. (Graphical units: 1 cm on the abscissa axis and 2 cm on the ordinate axis.)

1)

a- Determine the limit of f at $-\infty$.

b- Verify that $f(x) = \frac{x}{e^{x}} + e^{-x}\ln(1+e^{-x})$. Determine the limit of f at $+\infty$.

c- Deduce that the curve (C) admits two asymptotes whose equations are to be determined.

2) Consider the function g defined, on $]-1 , +\infty[$, by: $g(x) = \frac{x}{1+x} - \ln(1+x)$.

a- Prove that, on $]0 , +\infty[$, the function g is strictly decreasing.

b- Deduce the sign of g(x) when $x > 0$.

3)

a- Express $f'(x)$ in terms of $g(e^{x})$.

b- Construct the table of variations of f.

4) Draw (C).

Part B

Let F be a function defined, on IR, by: $F(x) = \int_{0}^{x} f(t)dt$.

1) Study the sense of variations of the function F.

2)

a- Verify that $\frac{1}{1+e^{t}} = 1 - \frac{e^{t}}{1+e^{t}}$ and determine $\int_{0}^{x} \frac{dt}{1+e^{t}}$.

b- Deduce, using integration by parts, F(x).

c- Verify that $F(x) = x - \ln(1 + e^{x}) - f(x) + 2\ln 2$ and that $F(x) = \ln\left(\frac{e^{x}}{1+e^{x}}\right) - f(x) + 2\ln 2$.

3) Determine $\lim\limits_{x \to +\infty} F(x)$.

4) Determine $\lim\limits_{x \to -\infty} (F(x) - x)$. Give a graphical interpretation of your result.

Part C

Let (U_n) be the sequence defined by: $U_n = f(1) + f(2) + \ldots + f(n)$, where $n \in IN^*$.

1) Shade, on the graphical representation, a domain whose area, in unit of area, is U_4.

2) Determine the sense of variation of the sequence (U_n).

3) Prove that $f(k) \le \int_{k-1}^{k} f(t)dt$, where $1 \le k \le n$, then compare U_n and F(n).

4) Is the sequence (U_n) convergent? Justify.

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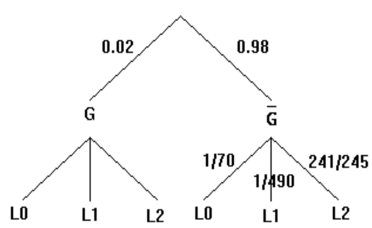
Answer Key

Question I								Mark
1)	**C**	**2)**	**B**	**3)**	**A**	**4)**	**B**	**0.5**
5)	**B**	**6)**	**B**	**7)**	**A**	**8)**	**C**	**each**

Question II			Mark
1)		$k = 2;\ \theta = \frac{2\pi}{3}$.	**0.25** **0.25**
2)	**a-**	$z' = \left(-1+i\sqrt{3}\right)z + i\sqrt{3}$.	**0.75**
	b-	$z_W = -\frac{3}{7} + \frac{2\sqrt{3}}{7}i$	**0.25**
3)	**a-**	$\overrightarrow{WE} = 2\overrightarrow{WA} - \overrightarrow{WB}$ and $\overrightarrow{WF} = \frac{2\overrightarrow{WA} + \overrightarrow{WB}}{3}$	**0.5** **0.5**
	b-	$\overrightarrow{WE} \cdot \overrightarrow{WF} = 0$; W moves on circle (C_1) of diameter [EF]	**0.75**
	c-	$WA = \frac{2\sqrt{7}}{7}$; W moves on circle (C_2) of center A and radius $\frac{2\sqrt{7}}{7}$.	**0.5** **0.5**
	d-	$\{W\} = (C_1) \cap (C_2)$.	**0.25**
4)	**a-**	$h(B) = C$ and $h(A) = A'$; $(A'C) // (AB)$; A' moves on the line passing through C and parallel to (AB). In addition, $h(A) = A'$; H, A, A' are collinear; A' moves on (HA). A' is the common point between these two lines.	**0.75**
	b-	ABC right and [AH] altitude; $AB \times AC = AH \times BC$; $AH = \sqrt{3}$. AHB right; $HB = 1$. $HC = BC - BH = 3$. Ratio of h $= \frac{HC}{HB} = 3$. $h \circ S\left(?\,,\ 2\times 3\,,\ \frac{2\pi}{3}\right)$; $h \circ S\left(?\,,\ 6\,,\ \frac{2\pi}{3}\right)$.	**0.75**

<table>
<tr><th colspan="3">Question III</th><th></th><th>Mark</th></tr>
<tr><td>1)</td><td colspan="2">$P\left(L_0 / \overline{G}\right) = \frac{241}{245}$.</td><td rowspan="2">
</td><td>0.5</td></tr>
<tr><td rowspan="3">2)</td><td>a-</td><td>$P(L_1) = P(L_1 \cap G) + P(L_1 \cap \overline{G})$;
$P(L_1) = P(L_1 / G) \times P(G) + P(L_1 / \overline{G}) \times P(\overline{G})$;
$0.016 = P(L_1 / G) \times 0.02 + \frac{1}{70} \times 0.98$; $P(L_1 / G) = 0.1$</td><td>1.5</td></tr>
<tr><td>b-</td><td colspan="2"><table><tr><th>$X = X_i$</th><th>-1000</th><th>9000</th><th>19000</th><th>T</th></tr><tr><td>$P(X = X_i)$</td><td>0.98</td><td>0.016</td><td>0.004</td><td>1</td></tr></table></td><td>0.5</td></tr>
<tr><td>c-</td><td colspan="2">$P(L_0) = 1 - (P(L_1) + P(L_2)) = 0.98$; $P(L_0) = P(L_0 \cap G) + P(L_0 \cap \overline{G})$;
$0.98 = P(L_0 / G) \times 0.02 + \frac{241}{245} \times 0.98$; $P(L_0 / G) = 0.8$.</td><td>1.5</td></tr>
</table>

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Question IV			Mark
1)		A ∉ (D); A and (D) determine a plane (Q). B(1 , 1 , 2) ∈ (D); $\overrightarrow{AM}\cdot\left(\overrightarrow{AB}\times\vec{u}_D\right)=0$; (Q): x + y – z = 0.	0.5
2)	a-	(Δ) ⊂ (P) and (Δ) ⊂ (Q).	0.5
	b-	A′(2 , t – 2 , t) ∈ (Δ); A′ orthogonal projection of A on (Δ) then $\overrightarrow{AA'}\cdot\vec{u}_\Delta=0$; t $=\frac{3}{2}$; $A'\left(2\ ,-\frac{1}{2}\ ,\frac{3}{2}\right)$.	0.5
3)	a-	H(0 , –1 , 2).	0.75
	b-	d(A , (P)) = 3. The angle is $\widehat{AMH}$; $\sin\alpha=\frac{HA}{AM}=\frac{d(A,(P))}{AM}=\frac{3}{AM}$; AM × sinα = 3.	0.25 0.5
	c-	α max.; AM min.; M ≡ A′; M is the orthogonal projection of A on (Δ). $\sin\alpha=\frac{HA}{AM}=\frac{3}{AA'}=\frac{\sqrt{6}}{3}$.	0.5
	d-	(AH) ⊥ (P), (AH) ⊥ (Δ); (AA′) ⊥ (Δ), then (Δ) ⊥ (A′H); α is the plane angle of the dihedral of the two planes (P) and (Q)	0.5
4)	a-	Radius = r = AA′ = $\frac{3\sqrt{6}}{2}$.	0.25
	b-	The center of (E) is the orthogonal projection of A on (P); center is H(0 , –1 , 2).	0.75
	c-	a = r = $\frac{3\sqrt{6}}{2}$; b = rcosα; $c^2 = a^2 - b^2 = r^2\sin^2\alpha$; c = 3; e = $\frac{\sqrt{6}}{3}$.	0.5
	d-	(F.A.) // (Δ) and passes through H, the center of (E); (F.A.): x = 0, y = k – 1, z = k + 2.	0.5

Question V			Mark
1)	a-	$S(C_1) = (C_2)$; ratio = $\frac{R_2}{R_1}$ = 2.	0.5
	b-	$\left(\overrightarrow{IB}\right)^2-4\left(\overrightarrow{IA}\right)^2=0$; $\left(\overrightarrow{IB}-2\overrightarrow{IA}\right)\cdot\left(\overrightarrow{IB}+2\overrightarrow{IA}\right)=0$; $-3\overrightarrow{IC}\cdot\overrightarrow{ID}=0$; $\overrightarrow{IC}\cdot\overrightarrow{ID}$ = 0. I moves on the circle of diameter [CD]; S(A) = B; I moves on the semi-circle of diameter [AB] deprived of A and B; I is the common point of the two sets.	1 0.5 0.5
2)		MB – MA = MQ + QB – (MP + PA) = 2 = constant = 2a; M moves on the hyperbola of foci A and B. 2a = 2; a = 1. 2c = AB = 9; c = 4.5 . b^2 = 84 / 4. e = 4.5 . Center is the midpoint of [AB]; center (–0.5 , 0). (H): $\frac{(x+0.5)^2}{1}-\frac{y^2}{\frac{85}{4}}=1$.	0.5 1.5
3)		h is a dilation of ratio $k_1 = \frac{R}{R_1}=\frac{R}{2}$; h′ is a dilation of ratio $k_2 = \frac{R_2}{R}=\frac{4}{R}$; h′ o h is a dilation of ratio k_1k_2 = 2 (≠ 1). h′ o h(C_1) = h′(C) = (C_2); h′ o h(A) = B; the center J of h′ o h is such that $\overrightarrow{JB}=2\overrightarrow{JA}$ or J ≡ C; h′ o h = H(C , 2).	1.5

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		Question VI	Mark
Part A			
1)	**a-**	$\lim\limits_{x\to-\infty} f(x) = 1$.	**0.5**
	b-	$f(x) = \frac{x}{e^x} + e^{-x}\ln\left(1+e^{-x}\right)$ (easy). $\lim\limits_{x\to+\infty} f(x) = 0$.	**0.5** **0.5**
	c-	y = 1 H.A. and y = 0 H.A.	**0.5**
2)	**a-**	$g'(x) = -\frac{x}{(1+x)^2} < 0$; g is strictly decreasing.	**0.5**
	b-	$g(0) = 0$ and g is strictly decreasing; $g(x) < 0$.	**0.5**
3)	**a-**	$f'(x) = \frac{1}{e^x} g\left(e^x\right)$.	**0.5**
	b-	Easy.	**0.5**
4)		Curve	**1**
Part B			
1)		$F'(x) = f(x) > 0$; F is strictly increasing.	**0.5**
2)	**a-**	$1-\frac{e^t}{1+e^t} = \frac{1}{1+e^t}$; $\int_0^x \frac{dt}{1+e^t} = \int_0^x \left(1-\frac{e^t}{1+e^t}\right)dt = \left[t-\ln\left(1+e^t\right)\right]_0^x = x - \ln(1+e^x) + \ln 2$.	**0.25** **1**
	b-	$F(x) = \int_0^x f(t)dt = \int_0^x \left(e^{-t}\ln\left(1+e^t\right)\right)dt = x - \ln(1+e^x) - e^{-x}\ln(1+e^x) + 2\ln 2$. ***(u = ln(1 + e^t) and dv = e^{-t}dx)***.	**1**
	c-	$F(x) = x - \ln(1+e^x) - f(x) + 2\ln 2$ and $F(x) = \ln\left(\frac{e^x}{1+e^x}\right) - f(x) + 2\ln 2$ (easy).	**0.25** **0.25**
3)		$\lim\limits_{x\to+\infty} F(x) = 2\ln 2$.	**0.5**
4)		$\lim\limits_{x\to-\infty} \left(F(x) - x\right) = -1 + 2\ln 2$; $y = x - 1 + 2\ln 2$ is an oblique asymptote to the curve of F.	**0.5** **0.25**
Part C			
1)		Easy.	**0.5**
2)		$U_{n+1} - U_n = \ldots = f(n+1) > 0$; (U_n) is strictly increasing.	**1**
3)		$k - 1 \le t \le k$ and f is decreasing; $f(t) \ge f(k)$; $f(k) \le \int_{k-1}^{k} f(t)dt$. $U_n = f(1) + f(2) + \ldots + f(n) \le \int_0^1 f(t)dt + \int_1^2 f(t)dt + \ldots + \int_{n-1}^{n} f(t)dt = \int_0^n f(t)dt = F(n)$; $F(n) \ge U_n$.	**1** **1**
4)		$U_n \le F(n) = 2\ln 2$; (U_n) is an increasing sequence that is bounded from above by $2\ln 2$; (U_n) is convergent.	**1**

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