I- (2 points)

Remark: The parts of this question are independent.

1) Solve the differential equation (E): $yy'' - 2y'^2 - y^2 = 0 \left(\text{Let } z = \dfrac{1}{y} \right)$.

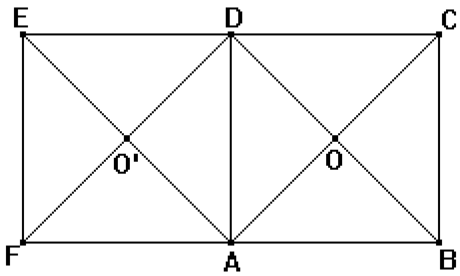
2) Find m so that the curve (C) of equation: $\dfrac{x^2}{m+1} + \dfrac{y^2}{2m-7} = 1$ is a rectangular hyperbola.

3) Let g be a function defined by: $g(x) = e^x + x^2$. Can you find a function f, other than g, so that $\lim\limits_{x \to +\infty} \dfrac{f(x)}{g(x)} = 1$? Justify your answer.

4) Calculate $\displaystyle\int_0^1 \frac{x}{x^4 + 4x^2 + 5} dx$.

II- (3 points)

In the adjacent figure, ABCD and ADEF are two identical direct squares. Consider:



- The rotation r of center A and angle $\dfrac{\pi}{2}$.
- The translation t of vector $\overrightarrow{AB}$.
- The dilation h of center C and ratio $\sqrt{3}$.

Let f = t o r and g = t o r o h.

1) Specify the image by f of the square ABCD.
2) Show that f is a rotation.
3) Prove that the point O is an invariant point under f.
4) Prove that g is a similitude whose angle and ratio are to be determined.
5) Let L be the center of g. Show that the triangle LDC is semi-equilateral and deduce a geometric construction of L.
6) The complex plane is referred to an orthonormal system $(A; \overrightarrow{AB}, \overrightarrow{AD})$.
   - **a-** Write the complex form of the similitude S that transforms A onto B and O onto E.
   - **b-** Specify S(C).

III- (2 points)

Given, in the space of an orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, the point A(1 , 1 , 1) and the two lines (D): $\begin{cases} x = t-1 \\ y = 2t+1 \\ z = -2t \end{cases}$ and (D'): $\begin{cases} x = 2m \\ y = -2m+3 \\ z = m-1 \end{cases}$ (t and m are two real numbers).

1) Prove that A does not belong to (D) and that (D') does not pass through A.
2) Write a cartesian equation of the plane that passes through A and parallel to (D) and (D').
3)
   a- Write a system of parametric equations of the straight line (L) passing through A and parallel to (D).
   b- Write a system of parametric equations of the straight line (L') passing through A and parallel to (D').
   c- Write a system of parametric equations of a bisector of the angle formed by (L) and (L').
4) Calculate cosine of the obtuse angle formed by (D) and (D').
5) Calculate the tangent of the acute angle formed by (D') and plane (xOy).

IV- (3 points)

Part A

An urn contains n red balls, 2n green balls, and 5 yellow balls, where n is a natural number greater than 1. Two balls are drawn successively without replacement from the urn. Let P(n) be the probability of choosing two balls having the same color.

1) Verify that p(n) = $\dfrac{5n^2-3n+20}{(3n+5)(3n+4)}$.
2) Find the number of the balls if p(n) = $\dfrac{4}{13}$.

Part B

In this part, assume that **n = 3**. A player selects, randomly and simultaneously, two balls from the urn. If the chosen ball is red, the player earns 5 points; if the chosen ball is green, he earns 3 points; if the chosen ball is yellow, he looses 4 points. Let X be the random variable that designates the sum of the points earned by the player (X could be negative).

1) Determine the probability distribution of X.
2) Calculate E(X).

Part C

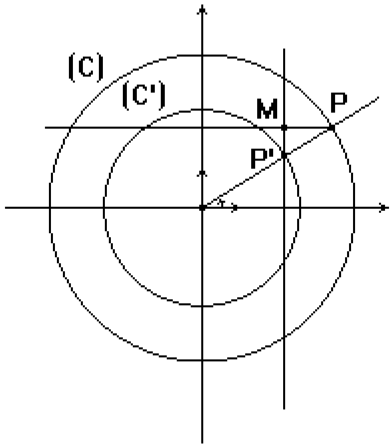
In this part, assume that **n = 4**. We select three balls, one after the other with replacement. Designate by Y the random variable that is equal to the number of the red balls obtained. Calculate P(Y = 2).

V- (3 points)

The plane is referred to a direct orthonormal system $(O; \vec{i}, \vec{j})$.

Part A

Let (C) be a circle of center O and radius $\sqrt{6}$ and (C') be a circle of center O and radius $\sqrt{2}$. Let P be a point on (C) such that $(\vec{i}, \overrightarrow{OP}) = \theta\ (2\pi)$. The segment [OP] cuts (C') at P'. The parallel to the axis of abscissas drawn from P and the parallel to the axis of ordinates drawn from P' intersect at M.



1) Prove that the coordinates of M are $(\sqrt{2}\cos\theta\,;\sqrt{6}\sin\theta)$.
2) Prove that, as θ varies on the interval]-π , π], the set of points M is an ellipse.

Part B

Let (E) be the ellipse of equation $3x^2 + y^2 = 6$.

1)
   - **a-** Draw (E) and the circle (C) on the answer sheet. (Unit: 2cm)
   - **b-** Determine the eccentricity, a focus, and the associated directrix of (E).

2)
   - **a-** Verify that the points $I(1, \sqrt{3})$ and $J(-\sqrt{2}, 0)$ belong to (E).
   - **b-** Determine the equations of the tangents (T) and (T') to (E) at I and J respectively.
   - **c-** Determine the coordinates of the point of intersection L of (T) and (T').
   - **d-** Let S be the midpoint of [IJ]. Prove that (LS) passes through the center of (E).

Part C

The objective of this part is to determine the nature of the curve (Γ) of equation $x^2 + y^2 + xy - 3 = 0$.

1)
   - **a-** Prove that (Γ) is the image of (E) by the rotation of center O and angle $\frac{\pi}{4}$.
   - **b-** Deduce that (Γ) is an ellipse whose center, axes and eccentricity are to be determined. Draw (Γ).

2)
   - **a-** Prove that (Γ) and (E) have the same auxiliary circle (C).
   - **b-** Calculate, in cm^2, the area of the domain bounded by (C) and (Γ).

VI- (7 points)

Consider the function f defined, on $\mathbb{R}$, by: $f(x) = 1 + e^{-x} - 2e^{-2x}$. Let (C) be the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

Part A

1) Find the limits of f at $+\infty$ and at $-\infty$.

2) Determine $f'(x)$, then construct the table of variations of f.

3) Let (D) be the straight-line of equation y = 1

a- Prove that (C) and (D) have one common point A whose coordinates are to be determined.

b- Study the relative positions of (C) and (D).

c- Determine an equation of (T), the tangent to (C) at A.

4) Draw (T), (D), and (C).

Part B

1) Calculate the area of the domain limited by (C), the ordinate axis, and (D).

2) Consider the sequence (U_n) defined, on $\mathbf{N}^*$, by: $U_n = \int_{(n-1)+\ln 2}^{n+\ln 2} (f(x)-1)\,dx$.

a- Show that $U_n > 0$.

b- Give a geometric interpretation of U_n.

3)

a- Using the sense of variations of f, show, for every $n \geq 2$, that:

$$f(n + \ln 2) - 1 \leq f(x) - 1 \leq f((n-1) + \ln 2) - 1 \text{ when } x \in [(n-1) + \ln 2 \,;\, n + \ln 2].$$

b- Deduce, for every $n \geq 2$, that: $f(n + \ln 2) - 1 \leq U_n \leq f((n-1) + \ln 2) - 1$.

c- Prove that the sequence (U_n) is decreasing.

d- Prove that the sequence (U_n) is convergent.

4) Consider the sequence (S_n) defined by: $S_n = U_1 + U_2 + \ldots + U_n$, where $n > 0$.
Write S_n in the integral form.

Answer Key

	Question I	Mark
1)	$y = \dfrac{1}{C_1 \cos x + C_2 \sin x}$.	**1**
2)	$m + 1 = -(2m - 7)$; $m = 2$.	**1**
3)	Yes; $f(x) = e^x$.	**1**
4)	$\dfrac{1}{2}\arctan 3 - \dfrac{1}{2}\arctan 2$.	**1**

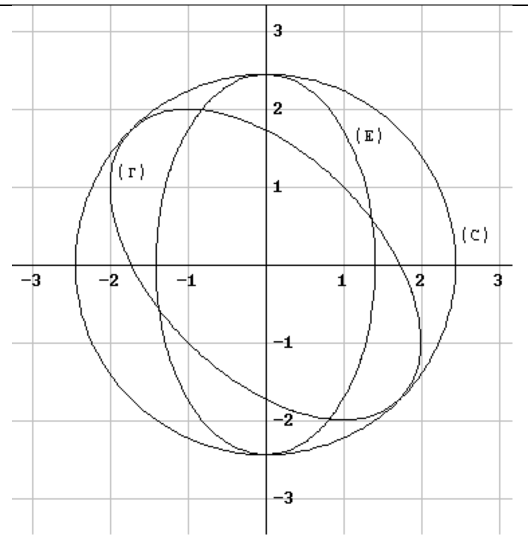
	Question II	Mark
1)	The image is the square BCDA.	**1**
2)	The composition of a translation and a rotation is a rotation.	**0.5**
3)	t o r(O) = t(O') = O.	**0.5**
4)	g = t o r o h = f o h, then g is a similitude being the composition of a dilation and a rotation; angle = $\dfrac{\pi}{2}$ and ratio = $\sqrt{3}$.	**1**
5)	g(C) = D, then $(\overrightarrow{LC}, \overrightarrow{LD}) = \dfrac{\pi}{2}$ and LD = $\sqrt{3}$ LC, then LDC semi equilateral and L is the third vertex of the direct semi equilateral triangle LDC.	**1**
6)	**a-** $z' = (-1 + 3i)z + 1$	**1**
	b- S(C) = C', where C'(–3 , 2)	**1**

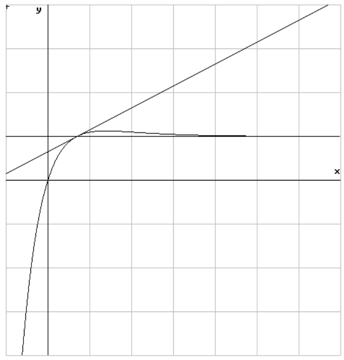
	Question III	Mark
1)	Easy.	**0.25** **0.25**
2)	$x + y + 3z - 4 = 0$.	**0.5**
3)	**a-** (L): $\begin{cases} x = k + 1 \\ y = 2k + 1 \\ z = -2k + 1 \end{cases}$ **b-** (L'): $\begin{cases} x = 2a + 1 \\ y = -2a + 1 \\ z = a + 1 \end{cases}$ **c-** (d): $\begin{cases} x = 3b + 1 \\ y = 1 \\ z = -b + 1 \end{cases}$	**0.5** **0.5** **1**
4)	Cosine is $-\dfrac{2}{9}$.	**0.5**
5)	Sine is $\dfrac{1}{3}$, then tangent is $\dfrac{\sqrt{2}}{4}$	**0.5**

Question IV		Mark
Part A		
1)	$p(n) = \dfrac{5n^2 - 3n + 20}{(3n+5)(3n+4)}$.	**1**
2)	n = 3	**0.5**
Part B		
1)	see table below	**3**
2)	$E(X) = \dfrac{169}{91}$	**0.5**
Part C		
$P(Y = 2) = \dfrac{624}{4913}$		**1**

$X = x_i$	-8	-1	1	6	8	10	Total
$P(X = x_i)$	$\dfrac{10}{91}$	$\dfrac{30}{91}$	$\dfrac{15}{91}$	$\dfrac{15}{91}$	$\dfrac{18}{91}$	$\dfrac{3}{91}$	1

Question V			Mark
Part A			
1)	$M\left(\sqrt{2}\cos\theta ; \sqrt{6}\sin\theta\right)$		**0.5**
2)	$\dfrac{x^2}{2} + \dfrac{y^2}{6} = 1$		**0.5**
Part B			
1)	**a-**		**0.5** **0.25**
	b-	$e = \dfrac{2}{\sqrt{6}}$; F(0 , 2); (d): x = 3	**0.5**
2)	**a-**	Easy	**0.5**
	b-	(T): $y = -\sqrt{3}x + 2\sqrt{3}$; (T'): $x = x_J = -\sqrt{2}$.	**0.5**
	c-	$L\left(-\sqrt{2}, \sqrt{6} + 2\sqrt{3}\right)$	**0.25**
	d-	(LS): $y = \left(-\sqrt{3} - \sqrt{6}\right)x$	**0.5**
Part C			
1)	**a-**	R(E) = (Γ)	**0.75**
	b-	Rotation preserves geometric figures; center O; $e = \dfrac{2}{\sqrt{6}}$; F.A.: y = −x	**0.5**
2)	**a-**	The two auxiliary circles have the same center and radius.	**0.5**
	b-	$\pi a^2 - \pi ab = (6\pi - 2\sqrt{3}\pi)u^2$	**0.25**



Question VI			Mark
Part A			
1)		1 and $-\infty$	**0.25** **0.5**
2)		$f'(x) = 4e^{-2x} - e^{-x}$; Easy.	**0.5** **1.5**
3)	**a-**	A(ln2 , 1)	**0.5**
	b-	x < ln2: (C) below (D); x > ln2: (C) above (D); x = ln2: (C) cuts (D)	**1**
	c-	(T): $y = \frac{1}{2}x - \frac{1}{2}\ln 2 + 1$.	**0.75**
4) (1.5 pts)			
Part B			
1)		Area = 0.5 u^2	**1**
2)	**a-**	When $x \in [(n-1) + \ln 2 ; n + \ln 2]$, $f(x) - 1 > 0$, then $U_n > 0$	**1**
	b-	U_n is the area of the domain bounded by (C), (D), $x = n - 1 + \ln 2$, and $x = n + \ln 2$.	**0.5**
3)	**a-**	On $[(n-1) + \ln 2 ; n + \ln 2]$; f is decreasing, $f(n + \ln 2) \le f(x) \le f((n-1) + \ln 2)$, $f(n + \ln 2) - 1 \le f(x) - 1 \le f((n-1) + \ln 2) - 1$.	**1**
	b-	Integrate	**1**
	c-	$U_{n+1} \le U_n$	**1**
	d-	(U_n) is decreasing and bounded from below (> 0), then (U_n) converges.	**1**
4)		$S_n = \int_{\ln 2}^{n+\ln 2} (f(x) - 1)\,dx$.	**1**