I- (2 points)

In the table below only one, among the proposed answers to each question, is correct. Write down the number of each question and give, **with justification**, the corresponding answer.

	Questions	Proposed Answers A	B	C
1)	BC = 8cm and $\hat{A} = 30^o$. The radius of the circle circumscribed about triangle ABC is	2 cm	4 cm	8 cm
2)	If $f(x) = x\arcsin x + \sqrt{1-x^2}$, then $f'\left(-\frac{\sqrt{2}}{2}\right) =$	$\frac{\pi}{4}$	$-\frac{\pi}{4}$	$-\frac{\sqrt{2}}{2}$
3)	$\int_{-1}^{1}(e^{x^2}+\sin^7 \pi x)dx =$	$2\int_{-1}^{1} e^{x^2}dx$	$2\int_{1}^{0} e^{x^2}dx$	$2\int_{0}^{1} e^{x^2}dx$
4)	$\int_{0}^{2}\frac{8dx}{x^2+4} =$	π	$\frac{\pi}{2}$	$\frac{\pi}{4}$

II- (3 points)

In the space of an orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the two lines (d_1) and (d_2) of parametric equations: (d_1): $\begin{cases} x = t-2 \\ y = 2t+1 \\ z = -2t+2 \end{cases}$ and (d_2): $\begin{cases} x = -2m \\ y = -4m+1 \\ z = 4m \end{cases}$, where t and m are two real numbers.

1)

a- Justify that (d_1) and (d_2) are parallel.

b- Let (P) be the plane formed by (d_1) and (d_2). Prove that a cartesian equation of (P) is $2x + y + 2z - 1 = 0$.

2)

a- Write a system of parametric equations of the straight-line (d) of (P), equidistant from (d_1) and (d_2).

b- Verify that an equation of plane (Q) that is perpendicular to (P) and contains (d), is: $2x - 2y - z + 5 = 0$.

3)

a- Let E be a point of (Q) that belongs to $(O;\vec{k})$. Verify that the coordinates of E are (0 , 0 , 5).

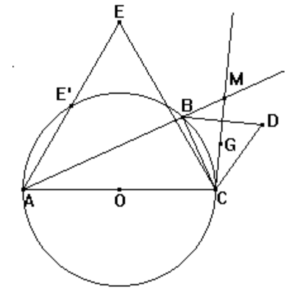
b- Let M be any point of (d_1). Verify that $EM^2 = 9t^2 + 12t + 14$.

c- Determine the coordinates of point H, the orthogonal projection of E on (d_2).

d- Deduce, from parts **b-** and **c-**, that E is equidistant from (d_1) and (d_2).

III- (3 points)

In the adjacent figure, (C) is a circle of center O and diameter [AC]. B is a point on (C) distinct from A and C. D and E are two points such that the two triangles BCD and ACE are direct and equilateral. G is the center of gravity of BCD. The two straight-lines (AB) and (CG) intersect at M. The straight-line (AE) cuts (C) at E'.



Part A

1) Prove that O, D, and G are on the perpendicular bisector (Δ) of [BC].
2) Let f be a dilation of center C that transforms (Δ) onto (AB). Determine the ratio of f and prove that G is the midpoint of [MC].
3) Let S be the similitude of center C that transforms B onto M.
   - **a-** Determine the ratio and the angle of S and prove that S(E') = E.
   - **b-** Let (C') be the image of circle (C) by S. Prove that the center O' of (C') is the point of intersection of (CE') and (OE).
   - **c-** Prove that (C') passes through C, E, M, and A. Draw (C').
   - **d-** Determine a rotation r and a dilation h such that r o h = h o r = S.

Part B

The plane is referred to a direct orthonormal system $(O;\vec{u},\vec{v})$ such that the affix of A is -1.

1) Determine the affix of each of C, E, and E'.
2)
   - **a-** Determine the complex form of each of f and S.
   - **b-** Determine the affix of O' and calculate the area of the circle (C').

IV- (2 points)

A player has a cubic die having six faces labeled from 1 to 6 and three urns U_1, U_2, and U_3 containing, each, k balls (k is a natural number and $k > 2$). There are three black balls in urn U_1, two black balls in urn U_2, and one black ball in urn U_3; all the remaining balls in the urns are white.

Consider the following game: The player rolls the die.

- If the obtained number is 1, then he chooses, randomly and simultaneously, two balls from U_1.
- If the obtained number is a multiple of 3, then he chooses, randomly and successively with replacement, two balls from U_2.
- If the obtained number is neither 1 nor a multiple of 3, then he chooses, randomly and successively without replacement, two balls from U_3.

Consider the following events:

N: "The chosen balls have different colors".

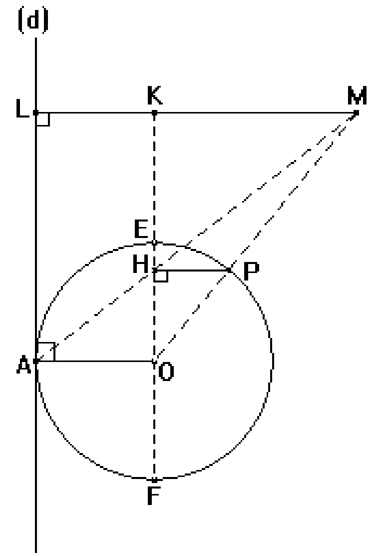
E_i: "The chosen urn is U_i" (i = 1, 2, or 3).

1) The player plays the above game.
   - **a-** Prove that $P(E_1) = \frac{1}{6}$, then calculate, in terms of k, $P(N \cap E_1)$.
   - **b-** Prove that the probability of choosing two balls of different colors is $\frac{10k^2 - 24k + 8}{3k^2(k-1)}$.
   - **c-** Calculate the probability that the die is labeled by 1, knowing that the chosen balls have different colors.
2) In this part, let k = 5. The player plays 20 games that are independent one from the other. Calculate the probability of choosing at least, one time, two balls having different colors.

V- (3 points)

In the adjacent figure,

- (C) is a circle of center O and radius 2 cm.
- A is a fixed point on (C) and P is a variable point on (C).
- The diameter of (C) perpendicular to (OA) cuts (C) at E and F.
- H is the orthogonal projection of P on (EF).
- M is the common point between (AH) and (OP).
- (d) is tangent to (C) at A.
- L is the orthogonal projection of M on (d).
- K is the common point between (EF) and (ML).



Part A

1) Prove that OM = $2\times\frac{KM}{PH}$.
2) Show that $\frac{MK}{ML}=\frac{HP}{OA}$ and deduce that ML = MO.
3) Specify the set (Γ) of points M as P varies on (C).

Part B

The plane is referred to a direct orthonormal system $(O;\vec{i},\vec{j})$, where $\vec{i}=-\frac{1}{2}\overrightarrow{OA}$.

1)
   - **a-** Verify that E and F belong to (Γ).
   - **b-** Determine the tangent to (Γ) at E.
   - **c-** Specify the vertex of (Γ) and draw (Γ).
   - **d-** Verify that an equation of (Γ) is $y^2 = 4(x + 1)$.
2) Let N be the point defined by $\overrightarrow{HN}=2\overrightarrow{HP}$.
   - **a-** Calculate the coordinates of point N in terms of the coordinates (x , y) of P and prove that N varies on an ellipse (E) of equation $\frac{x^2}{16}+\frac{y^2}{4}=1$.
   - **b-** Draw (E) in the same system of (Γ) and verify that E and F are the common points between (E) and (Γ).
3) Let (D) be the domain bounded by (Γ), (E), and the two straight-lines of equations x = –1 and x = 4. Calculate the volume of the solid obtained by the rotation of (D) around the axis of abscissas.

VI- (7 points)

Part A

Let (E) be the differential equation: $y' - 3y = \dfrac{-3e}{\left(1+e^{-3x}\right)^2}$. Let g and f be two defined functions, on $\mathbb{R}$, by: $f(x) = e^{-3x}g(x)$.

1) Write $g'(x) - 3g(x)$ in terms of $f'(x)$.
2) Determine f so that g is a solution of (E) that verifies $g(0) = \dfrac{e}{2}$.

Part B

Consider the function f defined, on $\mathbb{R}$, by: $f(x) = \dfrac{e^{1-3x}}{1+e^{-3x}}$. Let (C) be the representative of f in an orthonormal reference $\left(O;\vec{i},\vec{j}\right)$. Graph unit: 2cm.

1) Determine the limits of f at $-\infty$ and at $+\infty$, then study the variations of f.
2) Construct (C).
3) Let α be a given strictly positive real number. Suppose that $I_\alpha = \int_0^\alpha f(x)dx$.
   - **a-** Give the sign and a graphical interpretation of I_α in terms of α.
   - **b-** Express I_α in terms of α.
   - **c-** Determine the limit of I_α when α tends to $+\infty$

Part C

Let (U_n) be the sequence defined, on **N***, by: $U_n = \int_0^1 f(x)e^{\frac{x}{n}}dx$.

1)
   - **a-** Find the sign of U_n.
   - **b-** Determine the sense of variations of the sequence (U_n).
   - **c-** Is the sequence (U_n) convergent?

2)
   - **a-** Prove that $I_1 \leq U_n \leq e^{\frac{1}{n}}I_1$.
   - **b-** Deduce the limit of the sequence (U_n).

Answer Key

Question I								Mark
1)	**C**	**2)**	**B**	**3)**	**A**	**4)**	**A**	**1;1;1;1**

Question II			Mark
1)	**a-**	$\vec{U}_{d_2} = -2\vec{U}_{d_1}$	**0.5**
	b-	$(d_1) \subset (P)$ and $(d_2) \subset (P)$	**0.75**
2)	**a-**	$A(-2, 1, 2) \in (d_1)$; $B(0, 1, 0) \in (d_2)$; $I(-1, 1, 1)$ midpoint of [AB]; $I \in (d)$ and $\vec{U}_d = \vec{U}_{d_1}$; (d): $\begin{cases} x = \lambda - 1 \\ y = 2\lambda + 1 \\ z = -2\lambda + 1 \end{cases}$	**1**
	b-	$\overrightarrow{IM} \cdot (\vec{U}_d \wedge \vec{n}_P) = 0$	**0.75**
3)	**a-**	$E \in$ (z-axis), then $x_E = y_E = 0$; $E \in (Q)$, then $2x_E - 2y_E - z_E + 5 = 0$; $E(0, 0, 5)$	**0.5**
	b-	$M(t-2; 2t+1; -2t+2)$; $EM^2 = 9t^2 + 12t + 14$	**0.5**
	c-	$H \in (d_2)$, then $H(-2m; -4m+1; 4m)$; $\overrightarrow{EH} \cdot \vec{U}_{d_2} = 0$; $m = \frac{2}{3}$; $H\left(-\frac{4}{3}, -\frac{5}{3}, \frac{8}{3}\right)$	**1**
	d-	EM^2 is min. when $t = -\frac{2}{3}$; $EM = EH = \sqrt{10}$ u; E is equidistant from (d_1) and (d_2)	**1**

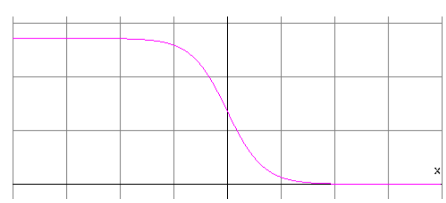
Question III			Mark
<u>Part A</u>			
1)		OB = OC (radii of (C)), then O belongs to (Δ). DB = DC (BCD equilateral), then D belongs to (Δ). GB = GC (G centroid of ABC, then G belongs to (Δ)	**0.25** **0.25** **0.25**
2)		f(O) = A and (Δ) is parallel to (AB), then ratio = 2. In the triangle CMA: $\frac{CO}{CA} = \frac{CG}{CM} = \frac{1}{2}$ then G midpoint [CM]	**0.25** **0.25**
3)	**a-**	Ratio $k = \frac{CM}{CB} = \frac{2\sqrt{3}}{3}$; angle $= (\overrightarrow{CB}, \overrightarrow{CM}) = -\frac{\pi}{6}$. CE′E is semi-equilateral, then S(E′) = E	**0.25** **0.25** **0.5**
	b-	Let O′ be the centroid of triangle ACE. COO′ is semi equilateral, then S(O) = O′	**0.5**
	c-	O′A = O′C = O′E (O′ is the centroid of triangle ACE), then A, C, and E belong to (C′); B ∈ (C), then S(B) ∈ S(C), then M ∈ (C′)	**0.25** **0.25**
	d-	$r\left(C; -\frac{\pi}{6}\right)$ and $h\left(C; \frac{2\sqrt{3}}{3}\right)$	**0.5**
<u>Part B</u>			
1)		$z_C = 1$; $z_E = i\sqrt{3}$; E′ midpoint [AE], then $z_{E'} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$	**0.25** **0.25**

2)	a-	f: $z' = 2z - 1$. S: $z' = \left(1 - i\frac{\sqrt{3}}{3}\right)z + i\frac{\sqrt{3}}{3}$	**0.5** **0.5**
	b-	S(O) = O′, then $z_{O'} = i\frac{\sqrt{3}}{3}$. Area (C′) = $\frac{4}{3} \times$ area (C) = $\frac{4\pi}{3}u^2$	**0.25** **0.5**

Question IV			**Mark**
1)	a-	$P(E_1) = \frac{1}{6}$ obtaining 1 out of six numbers; $P(N \cap E_1) = P(N/E_1) \times P(E_1) = \frac{k-3}{k(k-1)}$	**0.25** **0.5**
	b-	P(N)= $P(N \cap E_1) + P(N \cap E_2) + P(N \cap E_3) = \frac{k-3}{k(k-1)} + \frac{4(k-2)}{3k^2} + \frac{1}{k} = \frac{10k^2 - 24k + 8}{3k^2(k-1)}$	**2**
	c-	$P(E_1/N) = \frac{P(E_1 \cap N)}{P(N)} = \frac{3k^2 - 9k}{10k^2 - 24k + 8}$	**0.5**
2)		P(N when k = 5) = $\frac{23}{50}$; $P = 1 - \left(1 - \frac{23}{50}\right)^{20}$	**0.75**

Question V			**Mark**
Part A			
1)		In triangle OMK: $\frac{OM}{OP} = \frac{KM}{PH}$ and OP = radius = 2, then OM = $2 \times \frac{KM}{PH}$	**0.5**
2)		$\frac{MK}{ML} = \frac{MH}{MA} = \frac{MP}{MO} = \frac{HP}{OA}$, then $\frac{MK}{ML} = \frac{HP}{OA}$, then $\frac{MK}{HP} = \frac{ML}{2}$; OM = ML	**0.25** **0.25**
3)		(Γ) is a parabola of focus O and directrix (d)	**0.5**
Part B			
1)	a-	Let E′ be the orthogonal projection of E on (d), then EOAE′ is a square, then E ∈ (Γ). Similarly for F	**0.5** **0.5**
	b-	Tangent = (EA)	**0.25**
	c-	Vertex I is the midpoint of [OA]. Draw (Γ)	**0.25** **0.25**
	d-	I(−1 , 0) and O(0 , 0); $Y^2 = 4aX$; $y^2 = 4(x + 1)$	**0.5**
2)	a-	P(x , y) and H(0 , y); N(2x , y) ; P ∈ (C), then $x^2 + y^2 = 4$, then $\left(\frac{x_N}{2}\right)^2 + {y_N}^2 = 4$, then N moves on (E)	**0.5** **0.5**
	b-	Draw (E). $\begin{cases} y^2 = 4(x+1) \\ \frac{x^2}{16} + \frac{y^2}{4} = 1 \end{cases}$, then x = −16 (rej) and x = 0, then y = −2 or y = 2, then E(0 , 2) and F(0 , −2)	**0.25** **0.5**

3)	$V = \int_{-1}^{0} 4\pi(x+1)dx + \int_{0}^{4} \pi\left(4-\frac{x^2}{4}\right)dx = \frac{38\pi}{3}u^3$	0.5

		Question VI	Mark
Part A			
1)		$g'(x) - 3g(x) = e^{3x}f'(x)$	1
2)		g is a solution of (E), then $g'(x) - 3g(x) = \frac{-3e}{\left(1+e^{-3x}\right)^2}$, then $e^{3x}f'(x) = \frac{-3e}{\left(1+e^{-3x}\right)^2}$; $f'(x) = \frac{-3e\times e^{-3x}}{\left(1+e^{-3x}\right)^2}$; $f(x) = \int \frac{-3e\times e^{-3x}}{\left(1+e^{-3x}\right)^2}dx = e\times\int\frac{1}{u^2}du = -\frac{e}{1+e^{-3x}} + C$; $g(0) = f(0)$, $C = e$; $f(x) = -\frac{e}{1+e^{-3x}} + e = \frac{e^{1-3x}}{1+e^{-3x}}$	1.5
Part B			
1)		$\lim_{x\to-\infty} f(x) = e$; $\lim_{x\to+\infty} f(x) = 0$; $f'(x) = \frac{-3e^{1-3x}}{\left(1+e^{-3x}\right)^2} < 0$	0.5 0.5 1
1)		Variation table: x: $-\infty$ … $+\infty$; $f'(x)$: $-$; $f(x)$: decreasing from e to 0	1
2)			1.5
3)	a-	$f(x) > 0$ and $\alpha > 0$, then $I_\alpha > 0$. I_α is the area of the domain bounded by (C), the abscissa axis, and the two lines $x = 0$ and $x = \alpha$	0.5 0.5
	b-	$I_\alpha = \frac{e}{3}\ln\left(\frac{2}{1+e^{-3\alpha}}\right)$	1
	c-	$\lim_{\alpha\to+\infty} I_\alpha = \frac{e}{3}\ln 2$	0.5
Part C			
1)	a-	When $x \in [0, 1]$: $f(x) > 0$ and $e^{\frac{x}{n}} > 0$, then $f(x)\,e^{\frac{x}{n}} > 0$, then $U_n > 0$	0.5
	b-	$0 < n < n+1$; $\frac{1}{n} > \frac{1}{n+1}$; $x \geq 0$, then $\frac{x}{n} \geq \frac{x}{n+1}$; $\frac{x}{n+1} - \frac{x}{n} \leq 0$; $f(x) > 0$, then $\left(\frac{x}{n+1} - \frac{x}{n}\right)\times f(x) < 0$; $\int_0^1\left(\frac{x}{n+1} - \frac{x}{n}\right)\times f(x)dx \leq 0$; then $U_{n+1} - U_n \leq 0$, then (U_n) is decreasing	1.5
	c-	(U_n) is convergent because it is a decreasing sequence that is bounded below by 0	0.5
2)	a-	$0 \leq x \leq 1$; $n > 0$, then $\frac{0}{n} \leq \frac{x}{n} \leq \frac{1}{n}$; $e^{\frac{0}{n}} \leq e^{\frac{x}{n}} \leq e^{\frac{1}{n}}$; $1 \leq e^{\frac{x}{n}} \leq e^{\frac{1}{n}}$; $f(x) > 0$, then	1.5

		$f(x) \le f(x)e^{\frac{x}{n}} \le f(x)e^{\frac{1}{n}}$; $\int_0^1 f(x)dx \le \int_0^1 f(x)e^{\frac{x}{n}}dx \le \int_0^1 f(x)e^{\frac{1}{n}}dx$; $I_1 \le U_n \le e^{\frac{1}{n}} I_1$	
	b-	$\lim_{n\to+\infty} U_n = I_1 = \frac{e}{3}\ln\left(\frac{2}{1+e^{-3}}\right)$	**0.5**