Class: Third secondary: general Sciences

Subject: Mathematics

First Unified Exam

Exercise 1:(9pts)

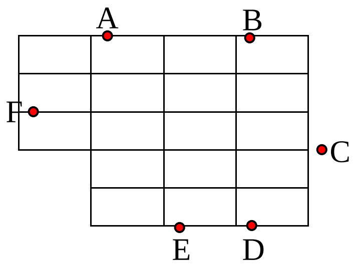
A) Each of the following questions has only one correct answer. Choose by justification the correct answers.

	Given	Question	Proposed answers		
			a	b	c
1	$f(x) = \left(2x - \frac{3}{x}\right)^7$	The coefficient of x^5 in f(x) is:	$3 \times 7 \times 2^6$	$-3 \times 7^2 \times 2^4$	$-3 \times 7 \times 2^6$
2	$z = \dfrac{(1+i\sqrt{3})^7 \times (1-i)^5}{(\sqrt{3}+i)^4 \times (1+i)^3}$	An argument of z is:	$\frac{5\pi}{6}$	$\frac{5\pi}{3}$	$\frac{\pi}{2}$
3	A and B are two events such that P(A) = 0.2 and P(B) = 0.4. Assume that $P(A \cap B) = 0.04$.	$P(\overline{A}/\overline{B}) = \dots$	$\frac{11}{15}$	$\frac{13}{15}$	$\frac{14}{15}$
4	f is a function defined on $[-3 ;+\infty[$ as $f(x) = -2 + \sqrt{3+x}$. g is the inverse function of f.	$g(x) = \dots$	$x^2 + 4x - 1$	$x^2 + 4x + 1$	$x^2 - 4x - 1$

B) Given the figure to the right.

1- Calculate the number of diagonals of the polygon ABCDEF.

2- Calculate the number of rectangles of the figure.



Exercise 2:(9pts)

In the complex plane referred to a direct orthonormal system $(O, \vec{u}, \vec{v})$, consider the points A, B and E of respective affixes $Z_A = 2i$, $Z_B = 1 + i$ and $Z_E = 3 + 4i$.

1- Find the exponential and the algebraic forms of $U = (Z_B)^{2010}$ and $V = (1 + Z_E)^5 \times \overline{Z_B}$.

2- Let (D) be the set of points M, of affixes z, such that : $|z - 2i| = |z - 1 - i|$.

a.Prove that (D) passes through the point E.

b. Show that (D) is a straight line.

3- Let (C) be the set of points M, of affixes z, such that: $|iz + 4 - 3i| = 3$.

a. Find the nature of (C).

b. Show that (D) cuts (C) at two points.

4- Find the coordinates of a point L such that the triangle LBE is right at L and LE = 2LB.

Exercise 3:(9pts)

An association organizes a fund raising dinner. The ticket prices are as follows:

- The ticket price for a children 10 years old or less is 5 € ;
- The price for a person between 11 and 16 years old is 8 € ;
- The ticket price for others is 10 €.

In addition, every member of the association benefits from a reduction of 20 % on the prices of the tickets so that, a member who is between 11and 16 years old will pay 6.4 €.

The participants in the event are distributed according to the table below:

Participant	Less than 10 years old	Between 11 and 16 years old	More than 16 years old	Total
Member	50	40	110	200
Non a Member	110	100	190	400
Total	160	140	300	600

At random, we choose a participant in the event.

1- What is the probability that he is a member of the association?

2- The chosen person is between 11 and 16 years old. What is the probability that he is a member of the association?

3- Consider the random variable X that is equal to the ticket price of a randomly chosen participant.

a. Verify that $P(X = 6.4) = \frac{1}{15}$.

b. Determine the possible values taken by X, and then find the probability distribution of X.

c. Determine, to the nearest 0.01, the mathematical expectation of X.

d. Estimate the total revenue for the association for the same tickets prices if the number of participants were 700.

<u>Exercise 4</u>: (9pts)

Let f be a function defined on R as $f(x) = x^5 + 2x^3 - 1$.

Let (C) be the representative curve of f in the plane of an orthonormal system $(O, \vec{i}, \vec{j})$.

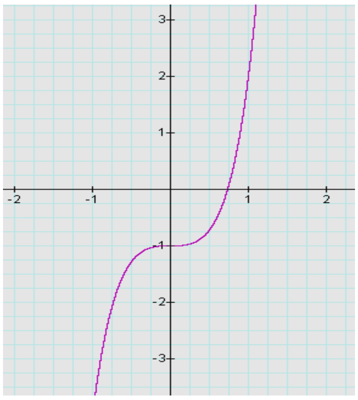
1-Show that f has over R an inverse function g and set up the table of variations of g.

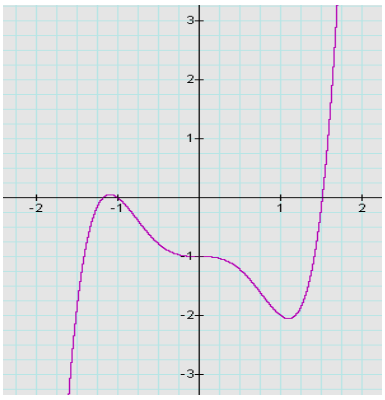
2-Let (C') designates the representative curve of g in the sysstem $(O, \vec{i}, \vec{j})$.

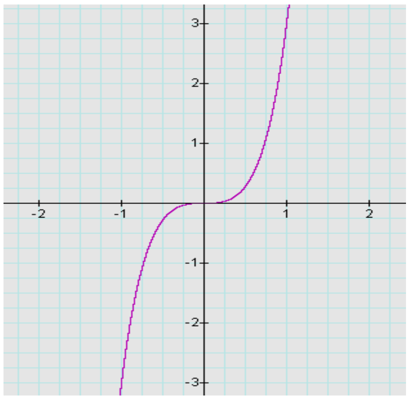
Verify that the point E (2; 1) is on (C') and write an equation of the tangent (L) through E to (C').

3-Show that (C') has only one point of inflection.

4- a. Among the curves below, choose the curve (C) and justify your choice.







b. Reproduce (C) and draw (C').

5- Consider $q(x) = \sqrt{1+x}$ and let G be a point of abscissa **0** on the curve $(C_{f \circ q})$.

Find an equation of the tangent through G to $(C_{f \circ q})$.

<u>Execise 5:(</u>4pts)

Consider the equation (E): $\arctan(2x) + \arctan(3x) = \frac{\pi}{4}$.

1. Using a calculator, check whether 2 is a root of (E).
2. Without using a calculator, check whether $\frac{1}{2}$ is a solution for (E).
3. Prove that the equation (E) has only one root α, frame α between two consecutive integers and then find an approximation for α to the nearest 0.1 by default.
4. Find the exact value of α

Exercise 1

A)1- $f(x) = \sum_{k=0}^{7} C_7^k (2x)^k (-3x^{-1})^{7-k} = \sum_{k=0}^{7} C_7^k (2)^k (-3)^{7-k} x^{2k-7}$

For 2k – 7 = 5 , k = 6 and the coefficient of x^5 becomes $C_7^6 \times 2^6 \times (-3) = -3\times7\times2^6$ ☞ (**c**)

2- $\arg(z) = 7\arg(1 + i\sqrt{3}) + 5\arg(1 - i) - 4\arg(\sqrt{3} + i) - 3\arg(1 + i) \pmod{2\pi}$

$= \frac{7\pi}{3} - \frac{5\pi}{4} - \frac{2\pi}{3} - \frac{3\pi}{4} \pmod{2\pi} = \frac{5\pi}{3} \pmod{2\pi}$ ☞ (**b**)

3- $P(\overline{A}/\overline{B}) = \frac{P(\overline{A}\cap\overline{B})}{P(\overline{B})} = ?$

$P(\overline{A}\cap\overline{B}) = P(\overline{A\cup B}) = 1 - P(A\cup B) = 1 - [P(A) + P(B) - P(A\cap B)] = \ldots = 0.44$

So, $P(\overline{A}/\overline{B}) = \frac{0.44}{0.6} = \frac{11}{15}$. ☞ (**a**)

4- $f(g(x)) = x \Leftrightarrow -2 + \sqrt{3+g(x)} = x \Leftrightarrow \sqrt{3+g(x)} = x + 2 \geq 0 \Leftrightarrow 3 + g(x) = (x+2)^2$

$\Leftrightarrow g(x) = x^2 + 4x + 1$ ☞ (**b**)

B) 1- $N = C_6^2 - 6 = 15 - 6 = 9$ diagonals.

2- $N' = C_5^2 \times C_4^2 + C_6^2 \times C_4^2 - C_4^2 \times C_4^2 = 60 + 90 - 36 = 114$ rectangles.

Exercise 2

1- $U = \left(\sqrt{2}\, e^{i\frac{\pi}{4}}\right)^{2010} = (2^{1005})e^{i\frac{2010\pi}{4}} = (2^{1005})e^{i\frac{\pi}{2}} = 2^{1005}i$

$V = \left(4\sqrt{2}\, e^{i\frac{\pi}{4}}\right)^5 \left(\sqrt{2}\, e^{-i\frac{\pi}{4}}\right) = 2^{13}e^{i(\frac{5\pi}{4}-\frac{\pi}{4})} = 2^{11}e^{i\pi} = -2^{13}$

2- a. $|z_E - 2i| = |3+2i| = \sqrt{13}$ and $|z_E - 1 - i| = |2+3i| = \sqrt{13}$ so $|z_E - 2i| = |z_E - 1 - i|$

(D) passes through E.

b. $|z_M - z_A| = |z_M - z_B| \Leftrightarrow MA = MB$

The set (D) is the perpendicular bisector of [AB] ; (D) is a straight line.

→→→OR (Popular way):

Let z = x + iy

$|z - 2i| = |z - 1 - i| \Leftrightarrow |x + i(y-2)| = |(x-1) + i(y-1)| \Leftrightarrow x^2 + (y-2)^2 = (x-1)^2 + (y-1)^2$

$\Leftrightarrow x - y + 1 = 0$

3- a. $|iz + 4 - 3i| = 3 \Leftrightarrow |z - 3 - 4i| = 3 \Leftrightarrow |z_M - z_E| = 3 \Leftrightarrow EM = 3$

(C) is a circle of centre E and radius R = 3.

→→→OR (Popular way)

b. The straight line (D) passes through the center E of (C), it cuts it at two points.

4- Use the Golden rule: $\frac{z_L - z_E}{z_L - z_B} = \frac{LE}{LB}e^{i\frac{\pi}{2}} = 2i \Leftrightarrow z_L - z_E = 2i\,(z_L - z_B)$

After calculation we get: $z_L = \frac{1}{5} + \frac{12}{5}i$.

Exercise 3 :

1- P(member of association) = $\frac{200}{600} = \frac{1}{3}$.

2-P(member of association/ between 11 and 16) = $\frac{40}{140} = \frac{2}{7}$.

3- a. $P(X = 6.4) = P(\text{ member between 11 \& 16}) = \dfrac{40}{600} = \dfrac{4}{60} = \dfrac{1}{15}$.

b. X takes the values: 4, 5, 6.4, 8 and 10.

x_i	4	5	6.4	8	10
p_i	$\frac{1}{12}$	$\frac{11}{60}$	$\frac{1}{15}$	$\frac{7}{20}$	$\frac{19}{60}$

c. E(X) = 7.65

We expect that the price of any ticket is almost 7.65 €.

d. The total revenue in case of 700 participants is : $700 \times 7.65 = 5355$ €.

Exercise 4:

1- f is a polynomial function, its continuous on R.

$f'(x) = 5x^4 + 6x^2 = x^2(5x^2 + 6) \geq 0$ and $f'(x) = 0$ for x = 0(isolated point), f is strictly increasing-

x	$-\infty$		0		$+\infty$
f '(x)		+	0	+	
f(x)	$-\infty$	↗			$+\infty$

f has an inverse function g

x	$-\infty$		0		$+\infty$
g '(x)		+	$+\infty$ ‖ $+\infty$	+	
g(x)	$-\infty$	↗			$+\infty$

2- Since f(1) = 2 so g(2) = 1 thus E(2 ; 1) is a point on (C').

$g'(2) = \dfrac{1}{f'(1)} = \dfrac{1}{11}$,

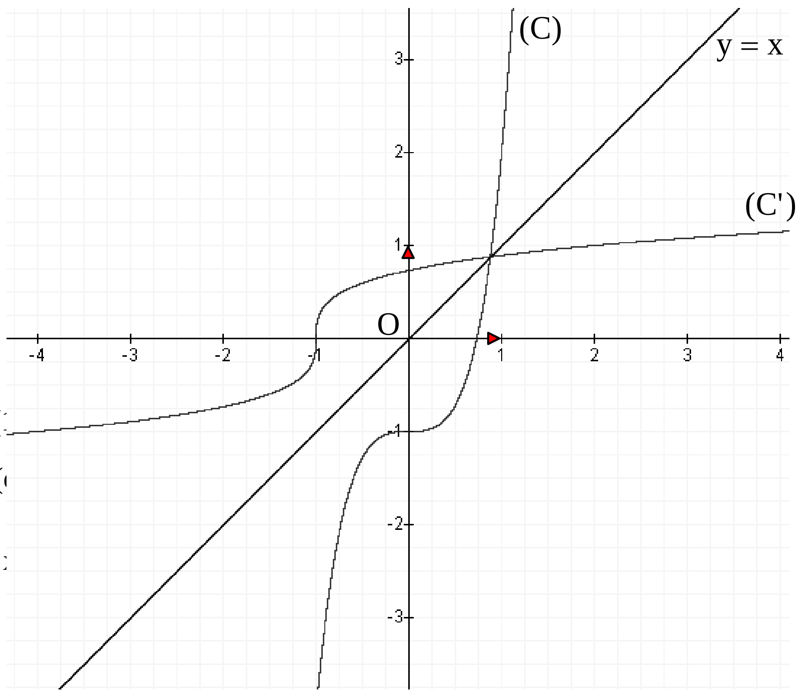
(T): $y - 1 = \dfrac{1}{11}(x - 2)$ so (T): $y = \dfrac{x+9}{11}$.

3- $f''(x) = 20x^3 + 12x = 4x(5x^2 + 3)$

f '' changes sign at x = 0 , so I(0 ; 1) is an inflection point of (C) and (C') has an inflection point I'(1 ; 0).

4-a. f is strictly increasing, we have two choices (1) or (3), choice 1 is true since f(0) = – 1.

b.



5- q(0) = 1 and f(

(f o q)' (0) = f ' (

(T) : $y - 2 = \dfrac{11}{2}$

.