Third secondary (ES section)

Mathematics(Final exam)

Exercise one: (4 points)

The table below shows the number of immigrants (in thousands) for the given year.

Year	2003	2004	2005	2006	2007	2008	2009
Rank of the year: x_i	0	1	2	3	4	5	6
Number (in thousands) of immigrants:y_i	4.9	5.1	5.3	5.4	5.6	5.75	5.81

a) Calculate the means $\bar{x} and \bar{y}$ of the variables x and y.

b) Represent graphically the scatter plot of the points (x_i, y_i) as well as the center of gravity G$(\bar{x}, \bar{y})$ in a rectangular system.

c) Calculate the linear correlation coefficient r and give an economical interpretation of the value thus obtained.

d) Determine the equation of $D_{y/x}$, the regression line of y in terms of x , and draw this line in the preceding system.

e) Suppose that the above pattern remains valid until the year 2015.

   $i)$ Estimate the number of immigrants of the given year.

   $ii)$ In which year does the number of immigrants exceed 10000?

Exercise two: (4 points)

On the day of their child's birth , Mr. and Mrs. Mabsout deposits 10,000,000 L.L. in a bank , paying an annual interest rate of 8% compounded annually with an addition of 400,000 L.L. added to the account at the end of each year .

Designate by C_0 the initial account $(C_0 = 10{,}000{,}000 L.L.)$ and by C_n the sum in Mabsout's account at the end of n^{th} year.

a) 1- Calculate C_1 and C_2

   2- Verify that $C_n = 1.08\, C_{n-1} + 400{,}000$

   3- Show that the sequence (C_n) is neither arithmetic nor geometric.

b) Suppose that for every natural number n : $u_n = C_n + 5{,}000{,}000$.

1- Show that (u_n) is a geometric sequence of common ratio 1.08.

2- Express u_n in terms of n.

3- What will be Mabsout's account at the end of 18^{th} year.

Exercise three: (4 points)

An urn U contains **12** balls: **7** red balls numbered from **1** to **7** and **five** black balls numbered from **8** to **12**. Three balls are drawn simultaneously and at random from the urn.

a) Find the number of all possible outcomes.

b) Consider the events: A: ''The three drawn balls have the same color''

B: ''The three drawn balls have even numbers ''

1- Verify that $P(A) = \frac{45}{220}$ *and calculate* $P(B)$.

2- Calculate $P(A \cap B)$ *then deduce* $P(A \cup B)$.

3- Are A *and* B *independent*?

c) To each red ball drawn you gain **1000L.L.** , and to each black ball drawn you lose **500 L.L.**Let **X** be the random variable that is equal to the total gain.

1- Verify that **X(Ω)** $= \{-1500, 0, 1500, 3000\}$.

2- Determine the probability distribution of **X** .

3- Calculate the expected value **E(X)** and the standard deviation of X.

Exercise four: (8 points)

Part A:

Consider the function f defined over $[0, +\infty[$ by: $f(x) = 1 - (x-2)e^{-x}$. Designate by (G) its representative curve in an orthonormal reference $\left(O; \vec{i}, \vec{j}\right)$ (Unit: 1 cm).

a- Show that $\lim_{x \to +\infty} f(x) = 1$. What does the line (d) of equation y = 1 represent for (G)?

b- Verify that $f'(x) = (x-3)e^{-x}$.

c- Set up the table of variations of f.

d- Verify that the line (T) of equation $y = -3x + 3$ is tangent to (G) at the point of abscissa zero.

e- Draw (d), (T), and (G).

f- Calculate the area bounded by (G), the axis of abscissas and the two lines of equations $x = 1$ and $x = 2$.

Part B:

The function M_c, defined over $[0, 8]$ by $M_c(x) = 1 - (x - 2)e^{-x}$, expresses the daily marginal cost of a factory for the production of pens, x being the number of pens expressed in hundreds, while $M_c(x)$ is expressed in millions L.L.

a- Knowing that the fixed cost amounts to 1 million L.L., verify that the total cost function C is expressed by:

$C(x) = x + 2 + (x - 1)e^{-x}$.

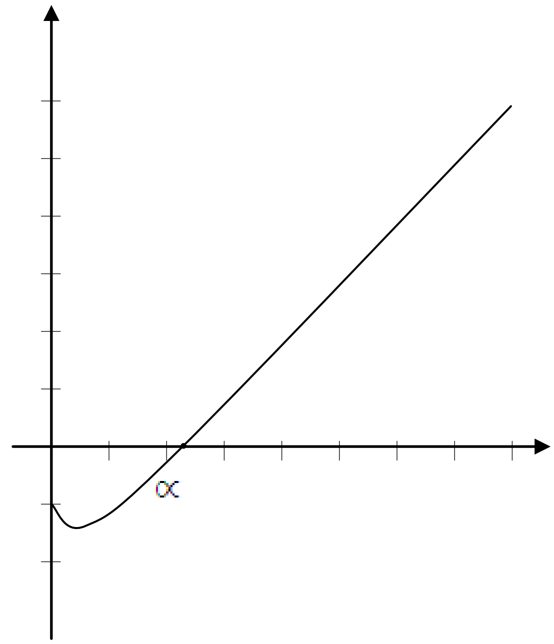
b- The pens are sold for 20 000 L.L. each, and suppose that the daily production is sold entirely. Show that the profit function P is expressed by:

$P(x) = x - 2 - (x - 1)e^{-x}$.

c-The adjacent curve represents the function P on [0 , 8]. Let α be the real number such that $P(\alpha) = 0$. Verify that:

$2.13 < \alpha < 2.14$

d-What is the minimum number of pens that should be daily produced by this factory, in order to realize a profit?



Exercise one:

a) $\bar{X} = 3 \ and \bar{Y} = 5.408$	0.5
b) Scatter plot	0.5
c) $cov(X,Y) = 0.62 , \sigma_X = 2, \sigma_Y = 0.3214$	

$r = 0.964$ near 1 indicates a strong positive correlation between X and Y	1.5
d) $D_{Y/X}: y = 0.155x + 4.943$	0.5
e) $i) x = 12 \rightarrow y = 6.8$ $ii)$ In the year 2036	0.5 0.5

Exercise two:

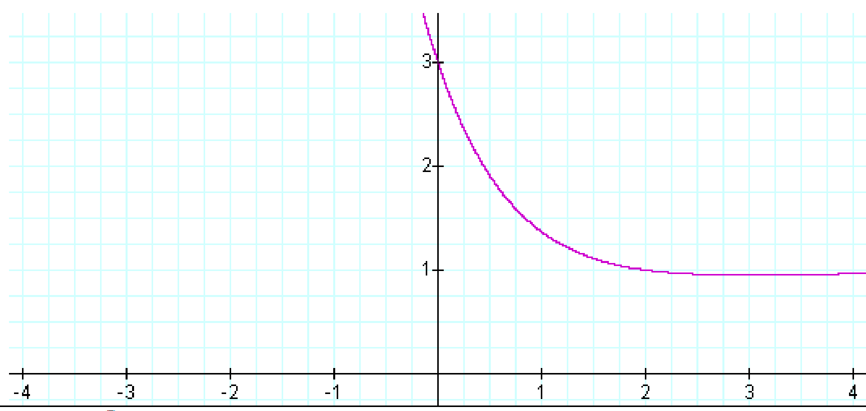
Part a:	
1) $C_1 = 11{,}200{,}000$, $C_2 = 12{,}496{,}000$	0.5
2) $C_n = C_{n-1} + \frac{8}{100}C_{n-1} + 400{,}000 = 1.08C_{n-1} + 400{,}000$	0.5
3) $C_1 - C_0 \neq C_2 - C_1$ so (C_n) is not arithmetic $\frac{C_1}{C_0} \neq \frac{C_2}{C_1}$ so (C_n) is not geometric	0.5 0.5
Part b:	
1) $\frac{u_{n+1}}{u_n} = 1.08$	1
2) $u_n = (10{,}000{,}000)(1.08)^n$	0.5
3) $u_{18} = 39{,}960{,}195$ $C_{18} = 34{,}960{,}195$	0.5

Exercise three:

a) $C_{12}^3 = 220$	0.25
b) 1-$P(A) = \frac{C_7^3}{C_{12}^3} + \frac{C_5^3}{C_{12}^3} = \frac{45}{220}$ $P(B) = \frac{C_6^3}{C_{12}^3} = \frac{1}{11}$ 2-$P(A \cap B) = \frac{C_3^3}{C_{12}^3} + \frac{C_3^3}{C_{12}^3} = \frac{1}{110}$ $P(A \cup B) = \frac{45}{220} + \frac{20}{220} - \frac{7}{220} = \frac{63}{220}$ 3-$P(A \cap B) \neq P(A) \times P(B) \rightarrow A$ and B not independent	0.5 0.5 0.25 0.25 0.25
c) 1-$BBB \rightarrow X = -1500$ $BBR \rightarrow X = 0$ $BRR \rightarrow X = 1500$ $RRR \rightarrow X = 3000$	0.5

2-$P(X=-1500)=\frac{1}{22}, P(X=0)=\frac{7}{22}$ $P(X=1500)=\frac{21}{44}$ $and\ P(X=3000)=\frac{7}{44}$	1
3-$E(X)=1125$ $V(X)=1342330 \to \sigma_X=1159$	0.5

Exercise four:

<table>
<tr><td>Part A:</td><td></td></tr>
<tr><td>a) $\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty} 1-\frac{x-2}{e^x}=1$</td><td>1</td></tr>
<tr><td>b) $f'(x)=-e^{-x}+(x-2)e^{-x}=(x-3)e^{-x}$</td><td>0.5</td></tr>
<tr><td>c)
$f'(x)=0\ iff\ x=3$
<table>
<tr><td>x</td><td>$-\infty$</td><td></td><td>3</td><td></td><td>$+\infty$</td></tr>
<tr><td>$f'(x)$</td><td></td><td>$-$</td><td></td><td>$+$</td><td></td></tr>
<tr><td>$f(x)$</td><td>$+\infty$</td><td>$dec.$</td><td>$1-e^{-3}$</td><td>$inc.$</td><td>1</td></tr>
</table></td><td>1</td></tr>
<tr><td>d) $y-f(0)=f'(0)(x-0)\to y=-3x+3$</td><td>0.75</td></tr>
<tr><td>e) 
</td><td>1</td></tr>
<tr><td>f) $A=\int_1^2 f(x)dx=2+e^{-2}\ unit^2$</td><td>0.75</td></tr>
<tr><td>Part B:</td><td></td></tr>
<tr><td>a) $C(x)=\int 1-(x-2)e^{-x}dx.\ \ and\ C(0)=0$
$then\ C(x)=x+2+(x-!)e^{-x}$</td><td>1</td></tr>
<tr><td>b) $P(x)=R(x)-C(X)=x-2-(x-1)e^{-x}$</td><td>1</td></tr>
<tr><td>c) $P(2.13)\times P(2.14)<0\to 2.13<\propto<2.14$</td><td>0.5</td></tr>
<tr><td>d) 214 pens</td><td>0.5</td></tr>
</table>