Class: Socio-Economics **Mid-Year Exam**
Subject: Mathematics

I- (4 points)

A factory manufactures shirts. The function g defined by: $g(x) = \frac{x}{2} + \frac{9}{2(x+1)}$ represents the marginal cost in millions of Lebanese pounds; x represents the number of shirts in hundreds, with $0 \le x \le 5$.

1) Calculate $g'(4)$. Give an economical interpretation of the obtained result.
2) Knowing that the fixed cost is null, prove that the total cost is $C_T(x) = \frac{x^2}{4} + \frac{9}{2}\ln(x+1)$.
3) Find the total cost for the production of 400 shirts.
4) Each shirt sells for 20 000 L.L.
   - **a-** Show that the revenue function R is given by R(x) = 2x.
   - **b-** Find the profit function P.
   - **c-** Does this company achieve a positive profit upon selling 30 items? Justify your answer.

II- (5 points)

In the "Image and Sound" section of a grand store, a television and a DVD are in a promotion for one week. A customer who visited this section is interviewed.

- The probability that this customer buys the television is $\frac{3}{5}$.
- The probability that this customer buys the DVD, knowing that he bought the television is 0.7 .
- The probability that this customer buys the DVD, knowing that he did not buy the television is 0.1 .

Consider the following events:

T: "The customer buys the television."
D: "The customer buys the DVD."

1) Calculate the probability of the following events:
   - **a-** "The customer buys the two appliances."
   - **b-** "The customer buys the DVD."
   - **c-** "The customer didn't buy any of the two appliances."
   - **d-** "The customer buys at least one of the two appliances."
2) The customer bought a DVD. What is the probability that this customer buys the television?
3) Before the promotion, the television costs 750 000 L.L. and the DVD costs 300 000 L.L.
   During this week, the grand store makes:
   * 15% discount for buying one of the two appliances.
   * 20% discount for buying the two appliances.
   Let X be the amount of money paid by a customer.
   - **a-** Justify that the possible values of X are: 0; 255 000; 637 500; 840 000.
   - **b-** Find the probability distribution of each value of X.

III- (11 points)

Part A

The adjacent table is the table of variations of the function g defined by: g(x) = $\frac{a\ln x}{x} - b$, where a and b are two real numbers.

x	0		e		$+\infty$
$g'(x)$	‖	$+$	0	$-$	
$g(x)$	‖ $-\infty$	↗	$\frac{1}{e}-1$	↘	-1

1) Use the table of variations of g to prove that a = 1 and b = 1.
2) Justify that g(x) < 0.

Part B

Consider the function f defined, on $]0\,;+\infty[$, by: f(x) = $\frac{1}{2}(\ln x)^2 - x + 4$. Let (C) be the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Calculate $\lim\limits_{x\to 0^+} f(x)$ and deduce an asymptote to (C).
2) Prove that $\lim\limits_{x\to +\infty} f(x) = -\infty$, and calculate f(7) and f(10).
3) Prove that f ′(x) = g(x), then construct the table of variations of f.
4) Prove that the equation f(x) = 0 admits a unique solution α. Verify that 5.4 < α < 5.5 .
5) Draw (C).
6) Solve f(x) > 0.

Part C

A factory produces toys. The demand and supply functions are given, in hundreds of toys, by the two functions f and h respectively, where f(x) = $\frac{1}{2}(\ln x)^2 - x + 4$ and h(x) = 1.5x – 1.

(x is the price in thousands of L.L. and 1 ≤ x ≤ 5.)

1) Find the demand and supply for a price of 2000 L.L.
2) Find the price if the supply is 350 toys.
3)
   - **a-** Verify, graphically, that the two functions intersect at a point of abscissa β. Verify that 2.11 < β < 2.12 .
   - **b-** Given that β = 2.115, give an economical interpretation for the obtained value of β.
4)
   - **a-** Find the elasticity E(x) in terms of x.
   - **b-** Calculate E(2.115). Is the demand elastic? Interpret your result economically.

GOOD WORK

Mid-year Exam
Answer Key
Socio-Economics

Question 1

1) $g'(4) = 0.32$

   Economical interpretation: The cost of production of the $5^{th}\, 100\ shirts\ is\ \ 320000\ l.p$

2) $c(0) = 0$

   $c'(x) = g(x)$

   $then\ c(x) is\ the\ total\ cost\ Function.$

3) $400\ shirts; x = 4$

   $Total\ cost = c(4) \times 10^6 = 11242471\ L.p$

4) $a -\ \ revenue = (20000)(100x) = 2x\ million\ L.p$

   So, $R(x) = 2x$

   $b -\ \ p(x) = R(x) - c(x) = 2x - \frac{x^2}{4} - \frac{9}{2}\, Ln\,(x+1)$

   $c -\ \ 30\ items;\ \ x = 0.3$

   $p(0.3) = \ -0.603$

   $then, the\ Factory\ suffers\ a\ loss\ of\ 603000\ L.p$

Question 2:(5 pts)

1) $a -\ \ P(D \cap T) = P({}^{D}/_{T}) \times (T) = 0.42$

   $b -\ \ P(D) = P(D \cap T) + P(D \cap \overline{T}) = o.46$

   $c -\ \ P(\overline{D} \cap \overline{T}) = P(\overline{D \cup T}) = 1 - P(D \cup T) = 0.36$

   $d -\ \ P(D \cup T) = 1 - P(\overline{D} \cap \overline{T}) = 0.64$

2) $P({}^{T}/_{D}) = \frac{P(D \cap T)}{P(D)} = 0.913$

3) $a -\ \ For\ never\ buying:\ amount\ paid = 0 \qquad X = 0$

   $For\ buying\ only\ DVD: amount\ paid = \frac{85}{100}(30000) \quad X = 255000$

   $For\ buying\ only\ television: amount\ paid = \frac{85}{100}(750000)\ \ X = 637500$

   $For\ buying\ both:\ \ X = 840000$

   b-

$X = x_i$	0	255000	637500636	840000	Total
$P(x_i)$	0.36	0.04	0.18	0.42	1

Question3:

(part A):(1.75 pts)

1) $\lim_{x \to +\infty} g(x) = -1$ (b=1)

   $g(e) = \frac{1-e}{e}$ (a=1)

2) max value of g is $\frac{1-e}{e} < 0, then\ g < 0\ For\ x \in D_g$

(part B):(5.25 pts)

1) $\lim_{x \to 0^+} f(x) = +\infty$ (y – axis: V.A)

2) $\lim_{x\to+\infty} f(x) = -\infty$ f(7)= - 1.1 f(10)= - 3.3 3) $f'(x) = g(x) < 0$

f: *strictly decreasing function*

x	0^+		$+\infty$
$f'(x)$		$-$	
$f(x)$	$+\infty$	$\searrow$	$-\infty$

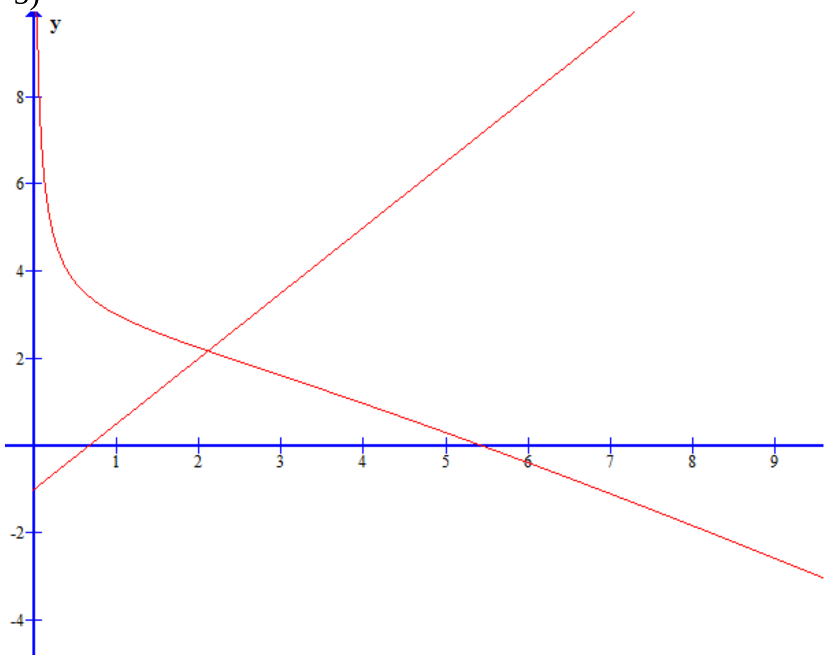
4) *f is continous and strictly decreasing From* $+\infty$ *to* $-\infty$ *so,* $f(x) = 0$ *has a unique soln* $\propto$

$f(5.4) = 0.02 > 0$

$f(5.5) = -0.04 < 0$

So, $\propto \in]5.4, 5.5[$

5)



6) $f(x) > 0$

(c): above x – axis

$x \in]0, \propto[$

(part c):

1) Price = 2000 L.L x=2
   Demand = 224 toys
   Supply = 200 toys
2) Supply = 350 toys, x = 3 price = 3000L.L
3) a. (c_f) *and* (c_h) *intersect at one point.*

   $f(2.11) > g(2.11)$

   $f(2.12) < g(2.12)$

   then $\beta \in]2.11, 2.12[$

   b. *For* $x = \beta = 2.115$, *there is a market equilibrium*

4) a. $E(x) = \frac{-xf'(x)}{f(x)}$

   b. $E(2.115) = 0.63 < 1$, *demand is inelastic.*

   Economical interpretation: For an increase of 1% in price , there is a decrease of 0.63% in demand.